The Price of Tail Risk in Liquidity and Returns

Lorán Chollete and Ching-Chih Lu*

January 19, 2012

Abstract

The likelihood of systemic risk presents a challenge for modern finance. In particular, it is important to know to what extent the market exacts a premium for exposure to 'tail risk'. In this paper, we use a simple estimate of two types of tail risk, in returns and liquidity, and measure their performance in a Fama and French (1993) style factor model. Empirically, *return* tail risk induces a monotonic pattern: stocks that are more sensitive to tail risk receive higher returns. Somewhat surprisingly, tail risk does not affect financial firms more than others. Tail risk exhibits relatively large returns and has very low correlations with other risk factors, suggesting that it represents a quite different type of risk. We document an economically and statistically significant premium between 1% and 3% for tail risk, which is robust to size, book-to-market, liquidity, downside risk, volatility and momentum. Furthermore, when we consider asset pricing tests, the only model to survive is one that augments the standard Fama-French model with a tail risk factor. Our results suggest that financial markets recognize tail risk in returns, which is reflected in the cross section of stocks. By contrast, *liquidity* tail risk is unpriced, which is a bit puzzling. When we estimate tail indices of liquidity and returns from high-frequency data, we discover they are always significantly correlated. This latter finding is consistent with the notion that episodes of tail risk in returns coincide with tail risk in liquidity.

Keywords: Asset Pricing; Financial Firms; High-Frequency Data; Liquidity Tail Risk; Systemic Risk

JEL Classification: C12, C32, G01, G12

---

*Chollete is at the University of Stavanger, email loran.g.chollete@uis.no. Lu is at National Chengchi University, email cclu@nccu.edu.tw. Chollete acknowledges support from the Research Council of Norway, Finansmarkedsfondet Grant #185339. This paper was previously circulated under the title "The market premium for dynamic tail risk". We are grateful for comments from Richard Davis, Paul Embrechts, Laurens de Haan, and Johan Segers, and from seminar participants at Columbia University, the Info-Metrics Institute at American University, UiS Business School, and the Fall 2011 Inquire Europe Conference "Risk and Returns after the Crisis".

1 Introduction

The financial crisis of 2008 called into question the ability of markets to deal with extreme events. A smoothly functioning financial system should award an appropriate premium to all relevant risks, such as large drops in prices and liquidity. A growing body of theoretical and policy research has treated the questions of systemic risk channels and optimal policy response.[1] However, there are few studies that tackle the issue of estimating the price required to compensate investors for exposure to systemic risk in asset returns and liquidity.

Systemic risk affects both financial markets and the real economy.[2] When extreme events occur in financial markets, propagation mechanisms may amplify their impact throughout the nation.[3] The demise of a major firm or lending institution evidently affects its customers, but may also have macroeconomic implications for aggregate consumption, investment, and unemployment.[4] For example, the 2008 collapse of Lehman Brothers damaged credit markets and scared employers and workers in all lines of business. Such magnified and correlated outcomes are vitally important for individuals in the economy, as well as for policymakers and investors. Moreover, from an academic viewpoint, extreme events are interesting, since they resemble results from a broad class of theoretical research on herding and strategic complementarities.[5] It is therefore valuable from several perspectives to obtain estimates of the effect of systemic risk on assets' required rates of return.

In light of these considerations, the main goal of this paper is to construct empirical estimates of the price of systemic risk. We therefore calculate measures of exposure to tail risk in US common stocks over the last half-century, and estimate the relevant risk premia. Intuitively, we should expect assets which tend to comove with systemic risk to be unattractive for risk averse investors to hold, after controlling for firm size,

[1] See Shin (2009); Acharya et al. (2010a); and Acharya et al. (2010c)

[2] For evidence on the welfare costs of extreme events, see Chatterjee and Corbae (2007), and Barro (2009).

[3] See Barro (2006) and Barro (2009). Also, see Horst and Scheinkman (2006), and Krishnamurthy (2010) for economic underpinnings of amplifications.

[4] For details on the macroeconomic importance of large firms, see Gabaix (2010a). For insurance during periods of economic disruptions, see Jaffee and Russell (1997); Jaffee (2006); and Ibragimov et al. (2009).

[5] See Wilson (1975); Bikhchandani et al. (1992); Cooper (1999); and Vives (2008), chapter 6.

liquidity and other factors. The reason is that they will be more expensive and difficult to sell when there is a large negative market-wide shock. Thus, they should carry a 'tail risk premium'. In this paper, we explore the conjecture that the cross-section of stock returns reflects a premium for bearing tail risk in liquidity and returns. A secondary goal of the paper is to assess the role of tail risk using high frequency data, and to investigate interactions between tail risk in returns and liquidity.

1.1 Related Literature

We build on previous research on asset pricing, systemic risk, and extreme economic events. Regarding asset pricing, Roy (1952) argues that investors care more about losses than gains. Kraus and Litzenberger (1976) develop a framework where individuals choose their investments based on a preference for positively skewed returns. Kahneman and Tversky (1979) show in a behavioral framework that agents may have loss averse preferences. Harvey and Siddique (2000) develop a model of conditional skewness in asset prices. They estimate that the premium for systematic skewness is significant and 3.6% per annum. Ang et al. (2006a) take the loss aversion concept to the data and examines whether stocks that covary with the market during market declines have higher average returns. They estimate the downside risk premium for the US to be approximately 6% per annum. In a related study, Ang et al. (2006b) conduct an empirical analysis of the effect of volatility on asset returns. They find that stocks with high exposure to aggregate volatility experience low returns, and that the volatility risk premium is approximately -1% per annum. The authors also document an important puzzle, namely, that high idiosyncratic risk stocks have exceptionally low returns. Patton (2006) shows that cash-constrained investors are better off when they account for skewness in asset returns. Regarding systemic risk, Danielsson and Zigrand (2008) construct a general equilibrium model where asset prices are determined in the presence of systemic risk. The authors argue that while regulation can reduce the likelihood of systemic risk, it carries costs, such as increased risk premia and volatility, and the possibility of non-market clearing. Acharya et al. (2010a) describe the causes of the financial crisis of 2008, arguing that a key catalyst was excessive leverage, which created systemic tail risk. Acharya et al. (2010b) construct a measure of systemic risk tendency, SES, based on comovement of expected shortfall of individual institutions and the aggregate financial system. They demonstrate the ex ante predictive power of SES for various companies during the period 2007-2009. Acharya et al. (2010c) develop an approach to regulating systemic risk based on SES. They propose that financial firms be taxed proportionally to their expected loss in the event of a systemic crisis. Bali et al. (2010) document high contemporaneous returns then low subsequent returns, for stocks that experience unexpected idiosyncratic volatility. The authors argue that this pattern is consistent with models of investor disagreement. Polson and Scott (2011) develop and test a model of cross-country contagion, based on common volatility shocks. On the theoretical side, researchers have established results that relate heavy tails, diversification and systemic risk. These results show that when portfolio distributions are heavy-tailed, not only do they represent limited diversification, they may drive a wedge between individual risk and systemic risk.[6] Thus, there are aggregate economic ramifications for heavy tailed assets, since individuals' diversification decisions yield both individual benefits and aggregate systemic costs. If systemic externality costs are severe, the economy may require intervention to improve resource allocation. These economic policy considerations do not seem to play a big role in most of the asset pricing work cited above. Moreover, none of the papers examines the empirical effect of tail risk on the cross section of stock returns. These issues provide an important motivation for our paper.

There is a large literature on extreme events and rare disasters in economics. Regarding extreme events, two early studies by Mandelbrot (1963) and Fama (1965) show that US stocks are not gaussian and have univariate heavy tails. Fama (1965) also documents that stock crashes occur more frequently than booms. Jansen and de Vries (1991) investigate the distribution of extreme stock prices using a univariate, nonparametric approach. They analyze daily data from ten $S\&P500$ stocks, and document that the magnitude of 1987's crash was somewhat exceptional, occurring once in 6 to 15 years. Susmel (2001) investigates the univariate tail distributions for international stock returns. He documents that Latin American markets have significantly heavier left tails than other industrialized markets. Susmel combines extreme value theory with the safety-first criterion of Roy (1952), and demonstrates improved asset allocation relative to the mean-variance approach. Longin and Solnik (2001) use a parametric multivariate approach to derive a general

[6] For evidence on limited diversification, see Embrechts et al. (2002) and Ibragimov and Walden (2007). For evidence on a wedge between individual risk and systemic risk, see Shin (2009); Ibragimov et al. (2009); and Ibragimov et al. (2011).

distribution of extreme correlation. The authors examine G5 equity index data to test for multivariate normality in both positive and negative tails. They document that tail correlations approach zero (consistent with normality) in the positive tail but not the negative tail. Further, Longin and Solnik (2001) show that correlations increase during market downturns. Hartmann et al. (2003) use an extreme value approach to analyze the behavior of currencies during crisis periods. Their results show that Latin American currencies have less extreme dependence than in east Asia, and that the developing markets often have a smaller likelihood of joint extremes than do the industrialized nations. Hartmann et al. (2004) develop a nonparametric measure of asset market dependence during extreme periods. The authors estimate the likelihood of simultaneous crashes in G5 stock and bond returns. Hartmann et al. (2004) document that stock markets crash together in one out of five to eight crashes, and that G5 markets are statistically dependent during crises. They conclude that the likelihood of asset dependence during extremes is statistically significant. Poon et al. (2004) use a multivariate extreme value approach to model the tails of stock index returns, in daily G5 stock indices. Poon et al. (2004) divide the data into several subperiods and country pairs, and document that in only 13 of 84 cases is there evidence of asymptotic dependence. They argue, therefore, that the probability of systemic risk may be over-estimated in financial literature. Longin (2005) develops hypothesis tests that differentiate between candidates for the distribution of stock returns, including the gaussian and stable Paretian. He then tests the distribution of daily returns from the $S\&$P500, and documents that only the student-t distribution and ARCH processes can plausibly characterize the data. Adrian and Brunnermeier (2010) build analyze a systemic risk measure, CoVaR, which summarizes the dependence of Value at Risk for different institutions, and represents the conditional likelihood of an institution's experiencing a tail event, given that other institutions are in distress. They estimate CoVaR for commercial banks, investment banks and hedge funds in the US. They document statistically significant spillover risk across institutions. Regarding rare events, Liu et al. (2003) analyze the role of rare events for asset allocation in a jump diffusion setting. They demonstrate that consideration of rare events discourages individual investors from holding leveraged positions. Related research by Liu et al. (2005) develops an equilibrium model of asset prices with rare events. The authors find that the equity premium comprises three parts, depending on risk aversion to jumps, aversion to diffusion movements, and aversion to uncertainty about rare events. The authors document that aversion to rare events can help ameliorate option mispricing. Barro (2006) builds a representative agent economy that incorporates the risk of a rare disaster, modelled as a large drop in the economy's wealth endowment. When this model is calibrated to the global economy, it can explain the equity premium and low risk free rate puzzles, and can help account for stock market volatility. Gabaix (2008), Gabaix (2010b), and Wachter (2011) generalize the Barro (2006) framework to account for dynamic probability of extreme events. These latter models are able to explain outstanding macroeconomic and finance puzzles as well as the behavior of stock volatility. Kelly (2011) estimates an average daily tail index from the cross section of stocks. The author shows that this measure predicts the aggregate market, and that stocks that are highly sensitive to this index earn low returns. Bollerslev and Todorov (2011) use high frequency options data to construct an index of implicit disaster fears among investors. This method is motivated by a jump-diffusion model that separates out disasters from smaller jumps in asset prices. The authors find that their method helps to explain patterns in the equity premium and stock market variance. These papers all underscore the importance of accounting for large, joint downward movements in asset returns. None of the papers, however, subjects the conjecture of an explicit price of systemic risk in liquidity and returns to empirical testing in a standard finance framework with tradable risk factors. This serves as a further motivation for our paper.

1.2 Contributions of Our Paper

We have 4 main contributions relative to the existing literature. First, we estimate a time series of daily tail risk in liquidity and returns in US stock markets. We then construct tradable risk factors TR and LTR, based on exposure to tail risk in returns and liquidity, respectively. Second, we analyze the pricing behavior of the two tail risks in the market. In particular, we explicitly compute risk premia and conduct asset pricing tests using both TR and LTR in a standard Fama-French framework. Third, we examine the relative exposure of financial companies to tail risk, as well as the relevance of leverage and book-to-market considerations. Finally, we use high-frequency data to document significant commonality between the tails of returns and liquidity.

More broadly, our research may yield practical insight into the functioning of the national economy where the 2008 crisis had its origins. Specifically, the results of our study help to address important academic and policy questions such as: What are the magnitude and price of exposure to tail risk in the US economy? Is tail risk in returns related to liquidity tail risk and other risk factors? Does tail risk affect Wall Street more than Main Street? Since we provide answers to these questions, our paper can contribute to the ongoing debate on financial regulation and market performance. Our paper is one of the first to analyze and explicitly price tail risk for liquidity and stock returns in the cross section, and to assess their empirical effects in a standard finance framework.

The remainder of the paper is organized as follows. In section 2 we outline the empirical content of our approach. In section 3, we describe the data and empirical results on computing aggregate tail risk indices. Section 4 presents the risk premia and asset pricing results for return tail risk. Section 5 discusses applications to liquidity tail risk and high frequency data, and Section 6 concludes.

2 Measuring Tail Risk

The goal of this project is to construct a proxy for tail risk in asset returns and liquidity, and then assemble portfolios of stocks based on exposure to tail risk. Based on these portfolios, we create tradable tail risk factors, which we use to compute risk premia, assess predictability, and conduct asset pricing tests.

2.1 Tail Indices and Power Laws

There are a number of estimates of systemic risk, which are based on the extreme value approach of quantile exceedances, or else power laws.[7] In either case, estimation focuses on a tail index, which assesses the likelihood of extreme events. Tail indices indicate whether asset returns have heavy tails, which have been

[7] For an extreme value approach, see Hartmann et al. (2003); Hartmann et al. (2004); and Acharya et al. (2010a). For a power law approach, see Gabaix et al. (2003).

theoretically linked to failure of diversification and systemic risk.[8] Tail indices relate to an important regularity in economics, that of power laws. Consider two variables X and C. Then following Gabaix (2009), we express a power law as a relation of the form $C = hX^{\alpha}$, for some unimportant constant h. The quantity α is called the power law exponent and controls extreme behavior of the particular distribution.[9]

Empirical estimation of heavy-tailedness is conducted using the concept of tail index, which is the same as the power law exponent above.[10] Assume that returns r_t are serially independent with a common distribution function $F(x)$. Consider a sample of size $T > 0$ and denote the sample order statistics as

$$r_{(1)} \leq r_{(2)} \leq ... \leq r_{(T)}.$$

Then the asymptotic distribution of the smallest returns $r_{(1)}$, written as $F_1(x)$, can be shown to satisfy

$$\begin{aligned} F_1(x) &= \left\{1 - \exp\left[-(1+kx)^{\frac{1}{k}}\right]\right\}, \text{ if } k \neq 0 \\ &= \left\{1 - \exp\left[x\right]\right\}, \text{ if } k = 0. \end{aligned} \tag{1}$$

The parameter k governs the tail behavior of the distribution. It is often more useful to examine the **tail index** α, defined as $\alpha = -1/k$. The distribution will have at most i moments, for $i \leq \alpha$. For example, if α is estimated to be 1.5, the data will only have well-defined means, but not variances. Thus, the smaller the tail index, the heavier the tails of the particular asset returns. We use the method of Hill (1975) to estimate the tail index.[11] The estimator, denoted α^H, is constructed as

$$\frac{1}{\alpha^H} = \frac{1}{q}\sum_{i=1}^{q}\left\{\ln|r_{(i)}| - \ln|r_{(q+1)}|\right\} \tag{2}$$

[8]See Embrechts et al. (2005); Ibragimov and Walden (2007); Ibragimov et al. (2009); and Ibragimov et al. (2011).

[9]For example, income research has documented that the proportion of individuals with wealth X above a certain threshold x satisfies the following relationship: $\Pr(X > x) \sim \frac{C}{x^{\alpha}}$, where $\alpha \approx 1$.

[10]The material on tail indices follows the exposition of Tsay (2002), and Gabaix and Ibragimov (2011).

[11]The Hill estimator is asymptotically normal, and consistent if q is chosen appropriately. For more details, see Tsay (2002); Embrechts et al. (2005); and de Haan and Ferreira (2006).

where q is a positive integer. Since we are interested in examining large negative returns, in our empirical implementation we choose a level of q that corresponds to the lower 5% tail of returns.[12]

Intuition for the Tail Index. The tail index measure in (2) assesses the average distance between the most extreme observations r_i and a benchmark r_{q+1}. Therefore, when this index is applied to the cross section of returns and liquidity, it varies monotonically with the average frequency of extreme realizations in the relevant dataset. For example, when applied to liquidity of various firms each day, equation (2) will be larger on days when more firms experience extremely low liquidity. This monotonic property with the likelihood of extremes is what makes the tail index an attractive empirical proxy for tail risk.

2.2 How Exposure to Tail Risk Affects Asset Prices

Intuitively, the risk of systemwide extreme events should affect risk averse investors' equilibrium demand for assets. We now discuss two alternative ways in which this intuition can be formalized, the standard discount factor framework (Campbell (2003); Ferson (2003)), and the dynamic rare disaster framework (Gabaix (2008); Wachter (2011).)

Implications from the Stochastic Discount Factor Framework. In an economy with no arbitrage, the first order condition for a representative agent holding a risky asset is

$$E_t[R_{i,t+1}, M_{t+1}] = 1, \tag{3}$$

where $R_{i,t+1}$ is the simple return on asset i and M_{t+1} is the agent's intertemporal marginal rate of substitution. M_{t+1} is called the pricing kernel, and prices risky asset payoffs.[13]

Expanding the expression in (3), we can write $1 = E_t[R_{i,t+1}M_{t+1}] = E_t[R_{i,t+1}]E_t[M_{t+1}]+Cov_t[R_{i,t+1}M_{t+1}]$. This implies

$$E_t[R_{i,t+1}] = \frac{1 - Cov_t[R_{i,t+1}, M_{t+1}]}{E_t[M_{t+1}]}.$$

[12]The cutoff of 5% is similar to that used by Gabaix et al. (2006); and Kelly (2011). Results with a 10% threshold are available upon request.

[13]See Lucas (1978); Harrison and Kreps (1979); Campbell (2003); and Ferson (2003).

Thus, an asset with high expected returns must have a relatively small covariance with the marginal rate of substitution. This type of asset will be very risky, because it does not deliver wealth during states of nature when the investor really needs wealth. The asset does not pay off during periods of high marginal utility, and will therefore need to have high returns, otherwise investors would not hold it.

Systemic risk presents a textbook case of a state of nature where investors have high marginal utility. For example, in the financial crisis of 2007-2009, many investors and home owners experienced dramatic declines in profits and income.[14] Consequently, risk-averse investors should demand higher returns for stocks that are highly correlated with a systemic risk factor. We apply this insight in the following section, by constructing tail risk factors corresponding to the pricing kernel M above[15] and computing the returns on stocks with different exposure to tail risk. Based on the reasoning above, we expect that stocks which are highly correlated with a systemic risk factor should have relatively large returns, and that there should be a positive premium for exposure to systemic risk.

Implications from the Dynamic Rare Disaster Framework. The recent work of Gabaix (2008), Gabaix (2010b) and Wachter (2011) underscores theoretical reasons for including dynamic tail risk in asset pricing models. A key insight from this body of research is that during extreme periods, fundamental asset values fall by an amount which varies over time. Such dynamic asset shortfalls result in time-varying risk premia and volatility. This framework provides a further theoretical basis to expect that assets with high exposure to tail risk should have larger required returns.

3 Data and Empirical Results on Tail Risk

Data are downloaded from CRSP. These data comprise common stocks listed on NYSE, AMEX and NASDAQ, which correspond to share codes equal to 10 or 11, and exchange code equal to 1, 2, or 3. The variables retrieved include returns, shares outstanding, price and trading volume in daily frequency. These

[14]For a summary of the causes and effects of the crisis, see Acharya and Richardson (2009); Brunnermeier and Pedersen (2009); and Acharya et al. (2010a).

[15]See the Appendix for details on the pricing kernel in an empirical framework.

daily variables are used to calculate the Hill estimators on the cross-section of stock returns, and to compute the average price impact as a liquidity measure for each day. For these calculations we apply a filter from \$5-\$1,000. All stocks with price exceeding \$1,000 or less than \$5 in the closing of the previous day are removed from the sample for only that day. The sample period is from July 1963 to December 2010. The starting date of 1963 is dictated by the availability of daily data for the Fama-French factors, which is July 1, 1963. After downloading this market data, we follow 3 steps: First, we construct Hill estimators of tail risks for asset returns and liquidity, denoted RTI and LTI for return tail index and liquidity tail index, respectively. This provides a daily series of tail risks in the market. Then we rank stocks into portfolios based on sensitivity to RTI and LTI, to construct monthly returns. Finally these portfolios are formed into a factor (5-1 differentials) as in the Fama and French (1993) framework. The factors are then used to estimate the price of tail risk in asset returns. We use the terms ”Tail Risk” (TR) and ”Liquidity Tail Risk” (LTR) to describe our risk factors.

3.1 Construction of Tail Index and Liquidity

We construct the raw tail risk index using the Hill estimator applied to the left tail of stock returns. Each day we estimate the left tail index α^H from equation (2) on the full sample of stock returns available at that day, using a benchmark of the lowest 5% of returns. The average monthly index is illustrated in Figure 1, and spikes around October 1987, August 1998 and October 2008. These periods correspond to important extreme events: the 1987 stock market crash, the LTCM crisis, and the collapse of Lehman Brothers, respectively. Therefore the tail index appears to reflect important periods of market turmoil. For robustness we also consider estimates from a 10% threshold, which are presented for daily data in Figure 2. Visually the two series are quite similar, and have a significant correlation of 0.90 (with a p-value less than 0.0001), which is reassuring for our methodology. We also examine the relationship between the tail index and volatility in daily and monthly data, in Figures 3 and 4. Volatility is measured by VXO, which assesses the implied volatility of a 30-day at the money option. VXO is available from the Chicago Board Options Exchange (CBOE). Visually the two series share some common spikes, especially in 1987, 1998 and 2008, although they appear less related at other periods. The volatility-tail index correlations for daily and monthly data are relatively small, at 0.13 and 0.16 (with p-values of 0.0001 and 0.0054), respectively. Therefore, the tail index may plausibly capture variation that is unrelated to volatility.

Since we control for liquidity risk, we need to construct a liquidity factor. Our liquidity measure of choice is that of Amihud (2002):

$$Liq_d^i = \frac{|r_d^i|}{Vol_d^i} \tag{4}$$

where Vol and r denote volume and returns, and Liq_d^i refers to the illiquidity of stock i on day d. We use this measure as the basis of our liquidity (tail and level) factors, based on evidence compiled by Goyenko et al. (2009). These authors compare different liquidity measures with high frequency measures as benchmarks, and document that the Amihud measure has the highest correlation with the benchmarks. We average (4) across all stocks each d in order to obtain a daily market illiquidity measure Liq_t, for use in the sensitivity regression below.

3.2 Tail Risk Factor TR

It is necessary to construct a risk factor, in order to perform asset pricing tests and compute risk premiums. We therefore follow a similar methodology to that developed by Fama and French (1993), and estimate annual risk loadings β^R and β^{liq} for each stock. These loadings are estimated while controlling for the Fama-French factors, and therefore are 'purged' of the effect of standard risk factors. Specifically, we sort stocks at the end of each June according to their betas for tail risk and illiquidity, estimated from the following time series regression

$$r_{it}^e = \beta^0 + \beta^M MKT_t + \beta^S SMB_t + \beta^H HML_t + \beta^{liq} Liq_t + \beta^R RTI_t + \varepsilon_{it} \tag{5}$$

where r_i^e and MKT represent excess returns on individual stocks and the market portfolio, SMB and HML are the Fama-French factors, and RTI is the tail index of cross-sectional daily returns, respectively. The regressions use daily data from July 1st of year $t-1$ to June 30th of year t. To reduce small sample bias in regression coefficients, we exclude all firm-year samples with less than 120 observations in any given year. Once we obtain the risk exposures β^{liq} and β^{R}, we use them to sort stocks into 5 quintile portfolios with an equal number of firms each June of year t, and hold a value-weighted portfolio, evaluated each month from July of year t to June of year $t+1$. Finally, we construct the liquidity and return tail risk factors as the difference between the portfolios with greatest and least sensitivity to liquidity, and to the return tail index, respectively. We denote these factors TR and LIQ, to capture tail risk in returns, and liquidity, respectively. For simplicity, we refer to return tail risk as just 'tail risk'. This procedure succeeds in extracting factors that measure stocks' exposure to tail risk, after controlling for standard risk factors.

Table 1 displays average factor returns for both tail risk portfolios. Interestingly, Panel A shows a decreasing *monotonic* pattern of returns across return tail risk portfolios. Economically speaking, the monotone pattern for tail risk means that stocks that are more sensitive to tail risk receive higher returns, which is suggestive of pricing importance. Moreover, tail risk has a statistically significant differential between the highest (RTI1) and lowest (RTI5) portfolios. In accordance with the asset pricing work of Fama and French (1993) and others, we term this differential the 'return tail risk factor', denoted TR. This factor has an economically significant value of nearly 5%. In economic terms, even after liquidity, market, size and book-to-market considerations, US investors that held stocks with high tail risk exposure required nearly 5% higher monthly returns than their counterparts who held stocks with low tail risk exposure. We will examine liquidity (level and tail) risk in more detail in Section 5. For the remainder of this section, we focus on return tail risk, TR.

How does the tail risk factor compare to other standard risk factors? To answer this question, we analyze average returns and correlations, in Table 2. The most striking result is that the tail risk factor's returns 0f 4.87 are higher than the Fama-French factors, and almost as large as the market return, 5.15. The largest correlation to tail risk is less than 0.14, so tail risk does not appear to be closely related to other factors.

3.3 Tail Risk in the Real and Financial Sectors

Does tail risk relate to the real economy? And does it tend to affect Wall Street or Main Street? We consider these questions below. Regarding tail risk and the real economy, we saw from Figure 1 that the tail index does not have a strong relation to the business cycle. A similar pattern is true for the TR factor: Figure 5 shows that TR is not strongly related to NBER recessions. This finding is economically intuitive if tail events happen randomly and are not systematically linked to productive activity of the real economy.

Regarding tail risk and Wall Street, we analyze the proportion of financial firms in TI portfolios every year, in Figure 6. This allows us to examine whether tail risk tends to be concentrated in financial firms. Evidently the percentage of financial firms does not differ systematically between low- (TI1) and high-exposure (TI5) portfolios. Moreover, there is no clear pattern regarding leverage or book-to-market ratios, as shown in Panels B and C. A general summary of these characteristics over the entire sample is presented in Table 3. Again, the most exposed firms do not tend to be financial firms, nor do they have higher leverage or book-to-market. Instead, there is a hump-shaped pattern, where the highest numbers of financial firms, leverage and book-to-market are for the middle portfolios TI3 and LIQ3.

To glean further insight on the role of Wall Street, we compute tail indices separately for all financial firms. The results are in Figure 7. The upper panel shows a marked difference between the two, especially in the early sample. Furthermore, the two series have only a modest correlation of 0.4. Thus there appears a significant difference between tail index of financials and other firms. As a final diagnostic, we present summary statistics in Table 4. The return tail index for financial firms has a modest correlation of -0.2 with Dow Jones returns, while the tail index for all firms is three times larger in absolute value. To summarize the results of this subsection, the average level of tail risk in financial firms and its relation to stock market returns appear to differ from that of the entire universe of stocks. However, the proportion of financial firms in the high tail risk quintiles is not systematically large year by year or over the entire sample.

3.4 Predictive Ability of Tail Risk

In order to investigate another potential linkage between tail risk and the economy, we conduct tests of causality and predictability. We consider two variables that are important from policy and academic perspectives, namely the yield spread and the market return.[16] Since the tests require stability in the data, we check for unit roots and stationarity, as presented in Table 5.[17] We are interested in monthly returns, so we focus on the monthly horizon. Yield spreads appear to have a unit root and be nonstationary, so inference involving yield spreads will be problematic. Therefore our following results on yield spread predictability are mainly for illustrative purposes.[18] Market returns, TR and the month-end tail index appear to be stationary and without a unit root, while the average monthly tail index appears to have a unit root. We therefore use the month-end tail index in the following causality and predictability tests.

We test causality with a 2-variable vector autoregression (VAR) framework, and present the results in Table 6. We focus on Panel B, since it is based on the more parsimonious BIC. The TR factor possesses highly significant information for future yield spreads but not for the market return, while the tail index has some information for future market returns. In the other direction, the market has information about future TR and tail index. As mentioned above, the nonstationarity of the yield spread requires us to be cautious about results on that variable. We now turn to formal predictability tests. Our framework is similar to that of Ang and Bekaert (2007), with results presented in Table 7. In Panel A, we present the results using TR as a predictor. The large standard errors around β^{TR} when predicting the market return indicate that TR has little predictive power for the market. However, TR has substantial predictive power for the yield spread at all horizons. A similar pattern exists for the tail index in Panel B. In general, our results indicate that tail index and tail risk cannot predict the market, although they can predict yield spreads. Somewhat surprisingly, the market return appears to have some information about future tail risk.

[16] Yield spread is the difference in returns between AAA and BAA bonds, and is a measure of default risk. These data are available from the Federal Reserve Bank of St. Louis.

[17] For more details on the unit root and stationarity tests, see Phillips and Perron (1988) and Kwiatkowski et al. (1992), respectively. Our application of the tests is close to that of Ang and Bekaert (2007), although we use longer horizon returns.

[18] We could difference yield spreads to remove the unit root, but this would prevent us from analyzing more than one-period ahead prediction.

4 The Pricing of Tail Risk

We now estimate the price of tail risk, and assess its performance in a standard finance setting. Our framework is a GMM-based linear factor model, as described in Cochrane (2005) and in the Appendix. Table 8 presents risk premia in the linear factor model framework. As explained in the Appendix, the risk premium measures the amount of return that an investor demands for a unit of exposure to the particular risk factor. Therefore the premium for tail risk measures the compensation to investors for holding stocks that have tail risk. We estimate risk premia using the CAPM and Fama-French (FF3) models, as well as these models augmented with liquidity LIQ and our tail risk factor TR. The test assets are the 5x5 size and book to market portfolios available from the website of Kenneth French.[19] The most important result is that tail risk is always significant. For example, even when it is added to the Fama-French 3 factor model, it receives a premium of more than 2%. This premium is more than double the magnitude of all other estimated premia, underscoring the importance of tail risk in the linear factor setting.

Formal asset pricing tests are presented in Table 9. J-stat is the Hansen (1982) test of over-identifying restrictions. HJ Dist is the distance metric of Hansen and Jagannathan (1997), which measures the maximum annualized pricing error for each model. Large p-values for the J-statistic and HJ distance indicate that the particular model fits well. The Delta-J test of Newey and West (1987) examines whether SMB and HML have additional ability to explain asset prices, relative to each alternative model. Small p-values for the Delta-J test indicate that addition of SMB and HML improves model fit. An explanation of these tests is in the Appendix. We use p-values of 0.05 as our cutoff levels for significance. In our table, the J-test and HJ-distance have their largest p-values for the Fama-French 3-factor model augmented by tail risk. Thus, the most plausible model is one that incorporates both Fama-French factors and tail risk. Turning to the Delta-J test, the relatively small p-values indicate that SMB and HML improve the fit of all models.

[19] We are grateful to Kenneth French for making these portfolios available.

4.1 Robustness

While the above results are highly suggestive, it is important to check for alternative explanations. Five plausible objections to our results have to with considerations of value-weighting liquidity, momentum, downside risk, volatility, and choice of data filter. Regarding value-weighting liquidity, our liquidity factor LIQ could possibly fail to capture liquidity effects by treating all firms equally. Consequently, large firms' true impact on market liquidity would be misrepresented. Moreover, in the last decade a large body of research has documented the importance of different liquidity measures in pricing the cross section of asset returns, for example Pastor and Stambaugh (2003); Acharya and Pedersen (2005); and Korajczyk and Sadka (2007); among others. We therefore construct a value-weighted version of the liquidity factor derived from (4). The results are presented in Table 10. Evidently, tail risk still receives significant premia in every specification. Indeed, the premia in the most comprehensive model (FF3 & VWLIQ & TR) slightly exceeds that from the corresponding model in Table 8. The asset pricing tests are very similar to those in Table 9: again the dominant model incorporates both Fama-French factors and tail risk.

Momentum is another candidate explanation, since stocks that are more sensitive to tail risk could be related to past winners. We therefore augment the above tests to include momentum considerations. Specifically, we use the momentum factor UMD of Carhart (1997) in the asset pricing tests. We also utilize the traded liquidity factor $PSLIQ$ of Pastor and Stambaugh (2003).[20] The results are presented in Table 11. The premium for TR is again significant in both specifications, and the best models include TR and Liquidity, according to both the J-test and the HJ distance. According to the Delta-J test's large p-values, SMB and HML do not add significant information to a model that includes CAPM, liquidity, momentum and tail risk.

Two important alternative explanations for our results are that tail risk could just be capturing downside comovement of stocks or systematic market volatility. We therefore examine robustness to downside risk and volatility. Table 12 estimates risk premia and asset pricing tests, where in addition to the regular market excess return, we include a downside market factor 'Down'. As in equation (13) of Ang et al. (2006a), this factor is equal to the minimum of the market return and the historical average. The risk premium for TR

[20] We are grateful to Lubos Pastor for making these data available on his website.

is still significant and the results are similar to those of our original tests. We now turn to a discussion of volatility. We construct the volatility factor FVIX of Ang et al. (2006b), and first present summary statistics in Table 13.[21] FVIX has relatively low returns at 0.54. These two factors are not closely related, as TR and FVIX have a relatively small correlation of around 4%. We present estimates of risk premia and asset pricing tests that include FVIX and TR in Table 14. The TR premium is again significant and larger than 2%, and the asset pricing tests suggest the best models should include TR.

Finally, our original estimation in (4) and (2) is based on the restriction that stocks be traded more than 120 days each years. This filter could arguably include very large or very small stocks, that are not necessarily representative of the market as a whole. We therefore apply an alternative filter, based on stock price. We restrict our data estimation to stocks that fall in the range \$5-\$1,000 from the previous year. Once a stock's closing price is outside this range, it is excluded from the cross-section sample until its price moves back to this range. Results are presented in Table 15. The estimated premia for tail risk are all significant. The only model to survive the J-test and HJ distance is one that augments the Fama French model with a tail risk factor. This result is therefore qualitatively the same as the original results in Table 9, without the filter.

To summarize, a measure of systemic risk of returns is priced in the US stock market. Moreover, our asset pricing tests show that the only model which cannot be rejected is typically one that contains the Fama-French factors as well as our tail risk factor. A CAPM model augmented with tail risk does not suffice to price the cross section of asset returns: SMB and HML, as well as momentum and volatility, generally contribute meaningfully to a model with CAPM and tail risk. Thus, although return tail risk is important, it appears to play a complementary role to existing factors.

[21]FVIX is constructed by projecting changes in the VIX index onto a set of base assets, as in Ang et al. (2006b), equation (4). Since 2003, CBOE changed the ”VIX” index used by Ang et al. (2006b) to VXO. VXO is highly correlated with the current VIX index. We therefore use VXO in calculating FVIX. Another reason for using VXO is that it is available back to January 1986 instead of January 1990 of the VIX index. This larger sample allows us to include the important extreme event of October 1987 in our sample.

5 A Liquidity Perspective on Tail Risk

Our preceding analysis focuses on tail risk in returns and its interaction with finance factors, in particular liquidity. This analysis can be profitably extended in at least two areas. First, tail behavior of liquidity is important, since dryups in liquidity are associated with business cycles, portfolio underdiversification, and financial crises (Brunnermeier and Pedersen (2009); Wagner (2011); and Odegaard et al. (2011)). Second, it is valuable to consider tail index estimation based on intraday data for individual stocks. Although the intraday approach does not have a long enough sample for standard asset pricing studies, it permits us to obtain a tail index for each stock, every day. These daily indices can be aggregated to form daily market indices, which are attractive because they are based on many observations each day. Such a market index is helpful to check how reasonable our cross-section based indices are, and to sharpen our intuition about tail risk through simple graphs and exploratory data analysis. We discuss these two perspectives on tail risk below, in turn.

5.1 Liquidity Tail Risk

We construct an estimate of the tail index in liquidity based on cross-section data, as we did earlier for returns. The liquidity measure is that of Amihud (2002), from equation (4). For purposes of comparison, we present the liquidity tail index along with the previous return tail index in Figure 8. The most striking finding is that the liquidity tail is generally below the return tail, which indicates a lower likelihood of extremes for liquidity, using the intuition from (2). The two tail indices have modest Pearson and rank correlations, at 0.26 and 0.23, respectively. Both correlations are strongly statistically significant, with p-values smaller than 0.0001. Intuitively, tail behavior of returns and liquidity is related, since the two tails co-move in a manner that is economically important.

Liquidity Tail Risk Factor As in Section 3 above, we construct a risk factor, in order to perform asset pricing tests and compute risk premiums. As before, we estimate annual risk loadings, β^L and β^{liq}, for each stock. These loadings are estimated while controlling for the Fama-French factors, and therefore are 'purged' of the effect of standard risk factors. Specifically, we sort stocks at the end of each June according to their betas for liquidity tail risk and illiquidity level, estimated from the following time series regression

$$r_{it}^{e} = \beta^{0} + \beta^{M} MKT_t + \beta^{S} SMB_t + \beta^{H} HML_t + \beta^{L} LTI_t + \beta^{liq} Liq_t + \varepsilon_{it} \tag{6}$$

where r_i^e and MKT represent excess returns on individual stocks and the market portfolio, SMB and HML are the Fama-French factors, and $Illiq_t$ is the illiquidity measure of Amihud (2002), and LTI is the tail index of cross-sectional daily liquidity. The regressions use daily data from July 1st of year $t-1$ to June 30th of year t.

Table 16 shows average returns on portfolios sorted on sensitivity to liquidity tail risk. Surprisingly, we find that there is very little dispersion across the portfolios. From Panel A, the difference in returns between stocks that are highly sensitive to liquidity tail risk and those that are not, is only 0.72 per cent per annum, and insignificantly different from zero. Liquidity level portfolios do not perform much better. Although this lack of return differential for extreme observations in liquidity may be due to our choice of liquidity measures, we still find it somewhat puzzling.

In order to conduct asset pricing exercises, we compute a liquidity tail risk factor "LTR" based on the 5-1 differentials, and present its summary statistics in Table 17. Panel A shows that, quite the opposite of return tail risk TR in Table (8) above, LTR has the lowest returns of all the factors. Panel B's correlations of LTR with other factors are low, beneath 15%. We turn to asset pricing tests in Table 18. From Panel A, we see that LTR never receives a significant premium. In Panel B, the results are again the opposite of their table 9 counterparts, where the largest models always had big p-values. This previous result does not obtain for liquidity tail risk: the J-statistic and HJ distance have minute p-values in all models. Further, the delta-J also has minute p-values, indicating that a model based on LTR and standard factors is inadequate. These results stand in broad contrast to those for return tail risk, and deepen the puzzle of nonpricing of liquidity tail risk.

5.2 Intraday-Based Tail Index

High-frequency data provides a useful setting for analyzing tail behavior in returns and liquidity. We construct intraday-based tail indices in both liquidity and returns in the following three steps. First, we calculate minute-by-minute returns and liquidity for each day from January 1993 to December 2010, using firms in the Trades and Quotes (TAQ) database.[22] These firms are filtered to include only those with a price between \$5 and \$1000 in the previous day. Given the variety of liquidity proxies in existence, we compute 4 liquidity measures: net order flow, effective spread, absolute spread and relative spread. Second, for each of these five series, we compute the Hill (1975) estimator from equation (2).[23] Third and finally, we average the firm tail indices to obtain an aggregate market tail index, for each day.

The high frequency-based tail indices are presented in Figures 9 to 13. There is evidence of nonstationarity in the tails for both returns and the spread measures, as shown in Figures 9 and 10: tail indices were very large before 1998, and then settled down to more moderate values subsequently. By contrast, Figure 13 shows that the final liquidity measure, net order flow, exhibited less dramatic shifts in the tails over the sample. Moreover, Figure 13 superimposes the cross-section based liquidity tail risk measure from Section 5.1 above. The cross-section based measure is almost always below the intrday measure, especially in recent years. Therefore, the cross section tail index may understate the true true magnitude of liquidity tail risk. This understatement might be an explanation for our finding of no pricing effects for liquidity.

How do the high-frequency based tails relate to each other? A basic answer to this question is provided in Table 19. Panel A presents standard Pearson correlations. Interestingly, the return tail is significantly correlated with all liquidity tails. However, Pearson correlations are notoriously fragile,[24], so Panel B presents the more robust Spearman or rank correlations. The most striking finding is that *all* tails are significantly correlated. For example, the rank correlation of return tails with all spread tails is always above 0.8! This

[22] For details on tail index estimation with high frequency data, see Dacorogna et al. (2001), chapter 5.

[23] Our estimates are based on the extreme 5% observations for each firm. We also compute a moment-based estimator of the tail index, as in de Haan and Ferreira (2006), chapter 3. These latter estimates are available from the authors upon request.

[24] For theoretical and empirical evidence on correlations versus robust dependence measures, see Embrechts et al. (2002); and Chollete et al. (2011), respectively.

provides quantitative evidence that historically more than 80% of the time, environments with extreme stock returns, also feature extremes in liquidity. In economic terms, this strong dependence between liquidity and return tails is consistent with the notion that episodes of tail risk in returns coincide with periods of tail risk in liquidity.

6 Conclusions

The recent financial crisis has underscored the importance of understanding and pricing systemic risk. Our research aims to deliver practical insight into the functioning of the national economy where the 2008 crisis had its origins. In particular, our study addresses the questions: What is the magnitude and price of exposure to tail risk in the US economy? Is return tail risk related to liquidity tail risk and other risk factors? Are financial firms more exposed to tail risk? We estimate a return tail risk premium of around 1 to 3%, tail risk is typically uncorrelated with other risk factors, and financial firms do not obviously suffer more tail risk. Since we provide answers to these questions, our paper may contribute to the ongoing debate on financial regulation and market performance. In our empirical approach we construct the average tail index for stock returns and liquidity as estimates of tail risk. We document that stocks have systematically higher returns if they are more exposed to return tail risk. We construct a tail risk factor TR, based on stock sensitivity to the tail risk series, net of market, liquidity, size and HML effects. TR exhibits larger average returns than other risk factors. It also has very low correlations with the other factors, suggesting that it may represent a quite different type of risk.

We document an economically and statistically significant premium for tail risk in returns (TR), but not for liquidity tail risk (LTR). The premium for TR is significant when tail risk is evaluated both in a CAPM and Fama-French 3-factor model. Furthermore, when we consider asset pricing tests, the overwhelmingly best model is one that augments the Fama-French model with a return tail risk factor. Our results are robust to alternative measures of liquidity, momentum, downside risk, volatility, and a price filter. By contrast, exposure to liquidity tail risk does not afford an extra risk premium, and the liquidity tail risk factor is unhelpful in a Fama-French pricing framework. Although this non-pricing of liquidity tail risk could be due to our measure for liquidity, it is somewhat puzzling.

An interesting finding of our research is comovement of tail indices in liquidity and returns. Based on daily data, the tail indices of liquidity and returns are modestly, significantly correlated, at 0.26. More telling, when tail indices for returns and liquidity are estimated from high frequency data, we confirm very strong, significant commonality between return tails and all liquidity tails. In particular, the rank correlation between return tails and spread tails always exceeds 80%. This latter finding suggests that episodes of tail risk in returns and liquidity coincide.

Surprisingly, exposure to tail risk does not seem to be concentrated in financial firms: the proportion of financial firms is roughly similar across the different portfolios. Thus, both Wall Street and Main Street had exposure to tail risk over the last half-century of US stock market history. Since return tail risk is empirically recognized by the market, academics and policymakers cannot assume that markets ignore the likelihood of extreme events. Our findings underscore the empirical relevance of tail risk for financial markets, and may justify further theoretical and empirical research on tail risk in the economy.

References

Acharya, A., Cooley, T., Richardson, M., Walter, I., 2010a. Manufacturing tail risk: A perspective on the financial crisis of 2007-2008. Foundations and Trends in Finance 4 (4), 247–325.

Acharya, A., Pedersen, L., 2005. Asset pricing with liquidity risk. Journal of Financial Economics 77, 375–410.

Acharya, V., Brownlees, C., Engle, R., Farazmand, F., Richardson, M., 2010b. Measuring systemic risk. In: Acharya, V., Cooley, T., Richardson, M., Walter, I. (Eds.), Regulating Wall Street: The Dodd-Frank Act and the New Architecture of Global Finance. John Wiley and Sons, New York, pp. 87–120.

Acharya, V., Pedersen, L., Philippon, T., Richardson, M., 2010c. A tax on systemic risk. In: Acharya, V., Cooley, T., Richardson, M., Walter, I. (Eds.), Regulating Wall Street: The Dodd-Frank Act and the New Architecture of Global Finance. John Wiley and Sons, New York, pp. 121–142.

Acharya, V., Richardson, M., 2009. Restoring Financial Stability: How to Repair a Failed System. John Wiley & Sons, New York.

Adrian, T., Brunnermeier, M., 2010. Covar: A systemic risk contribution measure. Tech. rep., Princeton University.

Amihud, Y., 2002. Illiquidity and stock returns: Cross-section and time-series effects. Journal of Financial Markets 5, 31–56.

Ang, A., Bekaert, G., 2007. Stock return predictability: Is it there? Review of Financial Studies 20 (3), 651–707.

Ang, A., Chen, J., Xing, Y., 2006a. Downside risk. Review of Financial Studies, 19 (4), 1191–1239.

Ang, A., Hodrick, R., Xing, Y., Zhang, X., 2006b. The cross-section of volatility and expected returns. Journal of Finance, 61 (1), 259–299.

Bali, T., Scherbina, A., Tang, Y., 2010. Unusual events and the cross section of asset returns. Tech. rep., Baruch College.

Barro, R., 2006. Rare disasters and asset markets in the twentieth century. Quarterly Journal of Economics 121 (3), 823–866.

Barro, R., 2009. Rare disasters, asset prices, and welfare costs. American Economic Review forthcoming.

Bikhchandani, S., Hirschliefer, D., Welch, I., 1992. A theory of fads, fashion, custom, and cultural change as informational cascades. Journal of Political Economy 100, 992–1026.

Bollerslev, T., Todorov, V., 2011. Tails, fears, and risk premia. Journal of Finance, forthcoming.

Brunnermeier, M., Pedersen, L., 2009. Market liquidity and funding liquidity. Review of Financial Studies 22 (6), 2201–2238.

Campbell, J., 2003. Consumption based asset pricing. In: Constantinides, G., Harris, M., Stulz, R. (Eds.), Handbook of the Economics of Finance, Volume 1B. Elsevier, New York, pp. 803–887.

Carhart, M., 1997. On persistence in mutual fund performance. Journal of Finance 52, 57–82.

Chatterjee, S., Corbae, D., 2007. On the aggregate welfare cost of great depression unemployment. Journal of Monetary Economics 54, 1529–1544.

Chollete, L., de la Pena, V., Lu, C., 2011. International diversification: A copula approach. Journal of Banking and Finance 35, 403–417.

Cochrane, J. H., 1996. A cross-sectional test of an investment-based asset pricing model. Journal of Political Economy 104 (3), 572–621.

Cochrane, J. H., 2005. Asset pricing, revised edition. Princeton University Press, Princeton, N.J.

Cooper, R., 1999. Coordination Games: Complementarities and Macroeconomics. Cambridge Press, New York.

Dacorogna, M., Gencay, R., Muller, U., Olsen, R., Pictet, O., 2001. An Introduction to High-Frequency Finance. Academic Press, New York.

Danielsson, J., Zigrand, J., 2008. Equilibrium asset pricing with systemic risk. Economic Theory 35, 293–319.

de Haan, L., Ferreira, A., 2006. Extreme Value Theory: An Introduction. Springer, New York.

Embrechts, P., McNeil, A., Frey, R., 2005. Quantitative Risk Management: Concepts, Techniques and Tools. Princeton Series in Finance. Princeton University Press, New Jersey.

Embrechts, P., McNeil, A., Straumann, D., 2002. Correlation and dependence in risk managament: Properties and pitfalls. In: Dempster, M. (Ed.), Risk Management: Value at Risk and Beyond. Cambridge University Press, Cambridge, UK, pp. 176–223.

Fama, E., 1965. The behavior of stock market prices. Journal of Business 38, 34–105.

Fama, E., French, K., 1993. Common risk factors in the returns on stocks and bonds. Journal of Financial Economics 33 (1), 3–56.

Ferson, W., 2003. Tests of multifactor pricing models, volatility bounds, and portfolio performance. In: Constantinides, G., Harris, M., Stulz, R. (Eds.), Handbook of the Economics of Finance. Elsevier, New York, pp. 743–802.

Gabaix, X., 2008. Variable rare disasters: A tractable theory of ten puzzles in macro-finance. American Economic Review: Papers and Proceedings 98 (2), 64–67.

Gabaix, X., 2009. Power laws in economics and finance. Annual Review of Economics 1, 255–293.

Gabaix, X., 2010a. The granular origins of aggregate fluctuations. Econometrica, forthcoming.

Gabaix, X., 2010b. Variable rare disasters: An exactly solved framework for ten puzzles in macro-finance. Tech. rep., New York University.

Gabaix, X., Gopikrishnan, P., Plerou, V., Stanley, H., 2003. A theory of power-law distributions in financial market fluctuations. Nature 423, 267–270.

Gabaix, X., Gopikrishnan, P., Plerou, V., Stanley, H., 2006. Institutional investors and stock market volatility. Quarterly Journal of Economics 121 (2), 461–504.

Gabaix, X., Ibragimov, R., 2011. Rank-1/2: A simple way to improve the ols estimation of tail exponents. Journal of Business and Economic Statistics 29 (1), 24–39.

Goyenko, R., Holden, C., Trzcinka, C., 2009. Do liquidity measures measure liquidity? Journal of Financial Economics 92, 153–181.

Granger, C., 1969. Investigating causal relations by econometric models and cross-spectral methods. Econometrica 37 (3), 424–438.

Hansen, L., 1982. Large sample properties of generalized method of moments estimators. Econometrica 50 (4), 1029–1054.

Hansen, L., Jagannathan, R., 1997. Assessing specification errors in stochastic discount factor models. Journal of Finance 52, 557–590.

Harrison, M., Kreps, D., 1979. Martingales and arbitrage in multi-period securities markets. Journal of Economic Theory 20, 381–408.

Hartmann, P., Straetmans, S., de Vries, C., 2003. A global perspective on extreme currency linkages. In: Hunter, W. C., Kaufman, G. G., Pomerleano, M. (Eds.), Asset Price Bubbles: Implications for Monetary, Regulatory and International Policies. MIT Press, Cambridge, pp. 361–382.

Hartmann, P., Straetmans, S., de Vries, C., 2004. Asset market linkages in crisis periods. The Review of Economics and Statistics 86 (1), 313–326.

Harvey, C., Siddique, A., 2000. Conditional skewness in asset pricing tests. Journal of Finance 55 (3), 1263–95.

Hill, B., 1975. A simple general approach to inference about the tail of a distribution. The Annals of Statistics 3 (5), 1163–1174.

Hodrick, R., 1992. ”dividend yields and expected stock returns: Alternative procedures for inference and measurement. Review of Financial Studies 5 (3), 357–386.

Horst, U., Scheinkman, J., 2006. Equilibria in systems of social interactions. Journal of Economic Theory 130, 44–77.

Ibragimov, R., Jaffee, D., Walden, J., 2009. Non-diversification traps in catastrophe insurance markets. Review of Financial Studies 22 (3), 959–993.

Ibragimov, R., Jaffee, D., Walden, J., 2011. Diversification disasters. Journal of Financial Economics 99, 333–348.

Ibragimov, R., Walden, J., 2007. The limits of diversification when losses may be large. Journal of Banking and Finance 31, 2551–2569.

Jaffee, D., 2006. Monoline restrictions, with applications to mortgage insurance and title insurance. Review of Industrial Organization 28, 83–108.

Jaffee, D., Russell, T., 1997. Catastrophe insurance, capital markets, and uninsurable risks. Journal of Risk and Insurance 64, 205–230.

Jansen, D., de Vries, C., 1991. On the frequency of large stock returns: Putting booms and busts into perspective. The Review of Economics and Statistics 73 (1), 18–24.

Kahneman, D., Tversky, A., 1979. Prospect theory: an analysis of decision under risk. Econometrica 47, 263–291.

Kelly, B., 2011. Tail risk and asset prices. Tech. rep., University of Chicago Booth School of Business.

Korajczyk, R., Sadka, R., 2007. Pricing the commonality across alternative measures of liquidity. Tech. rep., Northwestern.

Kraus, A., Litzenberger, R., 1976. Skewness preference and the valuation of risky assets. Journal of Finance 31, 1085–1100.

Krishnamurthy, A., 2010. Amplification mechanisms in liquidity crises. American Economic Journal-Macroeconomics 2 (3), 1–30.

Kwiatkowski, D., Phillips, P., Schmidt, P., Shin, Y., 1992. Testing the null hypothesis of stationarity against the alternative of a unit root. Journal of Econometrics 54, 159–178.

Liu, J., Longstaff, F., Pan, J., 2003. Dynamic asset allocation with event risk. Journal of Finance LVIII (1), 231–259.

Liu, J., Longstaff, F., Pan, J., 2005. An equilibrium model of rare-event premia and its implication for option smirks. Review of Financial Studies 18 (1), 231–259.

Longin, F., 2005. The choice of the distribution of asset returns: How extreme value theory can help? Journal of Banking and Finance 29, 1017–1035.

Longin, F., Solnik, B., 2001. Extreme correlation of international equity markets. Journal of Finance 56 (2), 649–76.

Lucas, R., 1978. Asset prices in an exchange economy. Econometrica 46 (6), 1429–1445.

Mandelbrot, B., 1963. The variation of certain speculative prices. Journal of Business 36, 394–419.

Newey, W., West, K., 1987. Hypothesis testing with efficient method of moments estimation. International Economic Review 28, 777–787.

Odegaard, B., Naes, R., Skjeltorp, J., 2011. Liquidity and the business cycle. Journal of Finance LXVI (1), 139–176.

Pastor, L., Stambaugh, R., 2003. Liquidity risk and expected stock returns. Journal of Political Economy 111 (3), 642–685.

Patton, A., 2006. Modelling asymmetric exchange rate dependence. International Economic Review 47 (2), 527–556.

Phillips, P., Perron, P., 1988. Testing for a unit root in time series regression. Biometrika 75, 335–346.

Polson, N., Scott, J., 2011. Explosive volatility: A model of financial contagion. Tech. rep., University of Chicago.

Poon, S., Rockinger, M., Tawn, J., 2004. Extreme value dependence in financial markets: Diagnostics, models, and financial implications. Review of Financial Studies 17 (2), 581–610.

Roy, A., 1952. Safety first and the holding of assets. Econometrica 20, 431–449.

Shin, H., 2009. Securitisation and system stability. Economic Journal 119, 309–322.

Susmel, R., 2001. Extreme observations and diversification in latin american emerging equity markets. Journal of International Money and Finance 20, 971–986.

Tsay, R., 2002. Analysis of Financial Time Series. John Wiley and Sons, New York.

Vassalou, M., Xing, Y., 2004. Default risk in equity returns. Journal of Finance LIX (2), 831–868.

Vives, X., 2008. Information and Learning in Markets: The Impact of Market Microstructure. Princeton Press.

Wachter, J., 2011. Can time-varying risk of rare disasters explain aggregate stock market volatility? Tech. rep., Wharton.

Wagner, W., 2011. Systemic liquidation risk and the diversity-diversification tradeoff. Journal of Finance, forthcoming.

Wilson, R., 1975. Informational economies of scale. Bell Journal of Economics 6, 184–195.

A Asset Pricing Tests

The asset pricing methodology we employ follows the stochastic discount factor (SDF) approach, similar to that of Cochrane (2005), and utilized by modern asset pricing studies. We specialize our analysis to a linear setting, in the tradition of Fama and French (1993).[25] We employ three different tests: the J-test, the HJ distance, the delta-J test and the supLM test. The basic setup may be expressed in the following manner. Consider an $n \times 1$ vector of gross returns R and a vector of asset prices p. Under conditions of no arbitrage there can be shown to exist a stochastic discount factor m, such that the following pricing relation holds:

$$E(Rm) = p \tag{7}$$

If we introduce a k-vector of risk factors f, and specialize to linear factor models, the relevant SDF is of the form

$$m = b_0 + f'b_1 \tag{8}$$

where b_0 is a constant, and b_1 is a k-vector of coefficients.[26] It can be demonstrated (Cochrane (1996); Ferson (2003)) that there is an equivalence between the linear discount factor of equation (8) and a factor pricing model expressed using factor risk premiums and betas. The equivalence can be expressed in the following manner, $E(R) = R^0 p + \beta' \lambda$. In this notation, the unconditional riskless rate is $R^0 = \frac{1}{b_0 + E(f')b_1}$, the vector of projections of asset returns on factors is denoted $\beta = cov(r, f')var(f)^{-1}$, and the risk premia may be calculated as

$$\lambda = -R^0 cov(f, f')b_1 \tag{9}$$

Now that we have displayed the basic framework we shall discuss the three tests in turn. First is the J test. We estimate the parameters b $= \{b_0 \; b_1\}$ by optimal GMM of Hansen (1982). From the data b is chosen to minimize the following objective function: $b = \arg\min J_t = g_T(b)'W g_T(b)$. Here g is defined as $g_T(b) = \frac{1}{T}\sum_{t=1}^{T} R_t y_t - p$, the vector of sample pricing errors, y is the candidate SDF, and W is the optimal weighting matrix. Hansen (1982) derives the distribution of the associated J-test statistic as

$$T * J_T \sim \chi^2(n-k) \tag{10}$$

where n is the number of orthogonality conditions and k is the number of parameters estimated.

One shortcoming of the J test is that it is model-specific; one might improve $J_t = g_T(b)'S^{-1}g_T(b)$ by inflating estimates of S rather than by lowering pricing errors g_T. Therefore we also consider a second test,the HJ distance of Hansen and Jagannathan (1997) To understand the HJ distance one can proceed in

[25] For further exposition of the SDF and linear factor model approaches, see Campbell (2003) and Ferson (2003), respectively. For asset pricing studies that apply these frameworks, see Ang et al. (2006b); and Vassalou and Xing (2004).

[26] The setup here closely follows that of Cochrane (2005), which derives the results in more detail.

the following fashion. Consider a proxy SDF y and the set of correct SDFs, M. The HJ distance δ is the minimum distance to the nearest correct SDF, and may be defined as

$$\delta = \min_{m \in L^2} \parallel y - m \parallel \tag{11}$$

subject to $E(mR) = p$ or, equivalently,

$$\delta^2 = \min_{m \in L^2} \sup_{\lambda \in R^n} E(y - m)^2 + 2\lambda'[E(mR) - p]. \tag{12}$$

Hansen and Jagannathan (1997) show that the solution to this program can be expressed as $\delta = [E(yR - p)'E(RR')^{-1}E(yR - p)]^{1/2}$, and that the estimation of the model's parameters can be cast in a GMM framework such that HJ distance is minimized. Empirically, this amounts to choosing b as

$$b = \arg\min \delta^2 = \arg\min g_T(b)'W_T g_T(b) \tag{13}$$

where $W_T = \frac{1}{T}\sum_{t=1}^{T}(R_t R_t')^{-1}$. This is the approach we use for constructing the HJ distance metric. Hansen and Jagannathan (1997) also note that the HJ distance can be interpreted as the maximum pricing error for the test portfolios, with (portfolio) return having a norm of unity.

The third test is the delta-J test. The delta-J test examines whether other risk factors (in this case, HML and SMB) have any additional explanatory power in the presence of the proposed models. Suppose we have two sets of factors, f_1 and f_2, and wish to determine whether the set f_2 is irrelevant in the presence of f_1. One method, akin to the classical Likelihood Ratio test, is to estimate both the unrestricted and restricted models, respectively, $m = b_1' f_1 + b_2' f_2$ and $m = b_1' f_1$, then compare the J test statistic defined in (10) above. The J statistic should be larger for the restricted case since there are fewer parameters to estimate. To assess whether the increase in the J statistic is significant, we utilize the delta-J statistic, which is distributed as

$$\Delta J = TJ_{\text{restricted}} - TJ_{\text{unrestricted}} \sim \chi^2(q) \tag{14}$$

where q is the number of restrictions. For example, in the context of our framework, f_1 can correspond to CAPM augmented with liquidity and the tail risk factor, and f_2 corresponds to HML and SMB.

Figure 1: Average Monthly Tail Risk in Asset Returns

The figure shows the tail index estimator of Hill (1975), applied to stock returns. The tail index is estimated from equation (2) in the cross-section of stocks every day to obtain a market tail risk index. The threshold level is the lowest 5% of the data. In this figure the market tail index is the averaged over all days in each month to obtain a measure of monthly market tail risk. The data comprise NYSE, AMEX and NASDAQ stocks with prices between $\$5$ and $\$1000$. Shaded areas denote NBER recessions. The sample period is 1964 to 2010.

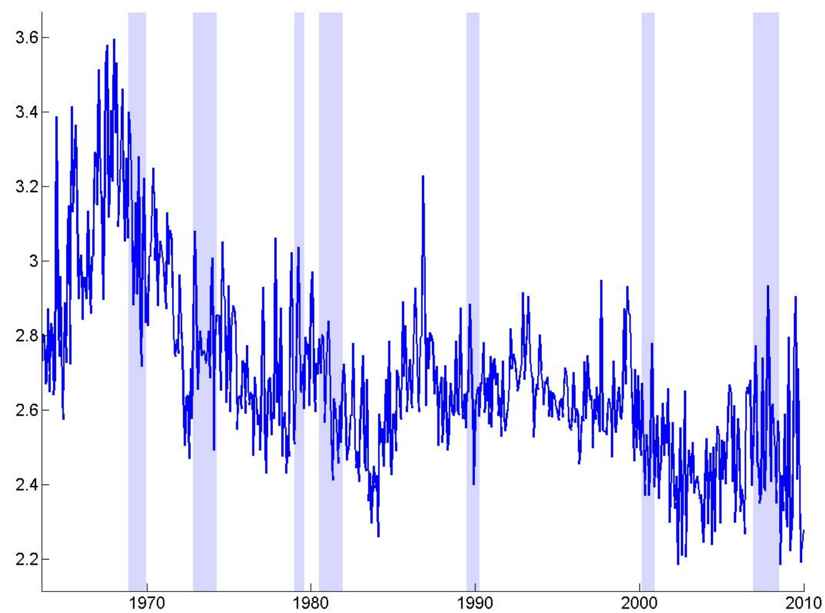

Figure 2: Daily Return Tail Index using Alternative Thresholds

The figure shows the return tail index estimated using the method of Hill (1975), from equation (2). We use both 10% and 5% thresholds, that is, we estimate the index using the lowest 10 and 5 percent of returns in the cross section of stocks each day, respectively. The data comprise NYSE, AMEX and NASDAQ stocks with prices between $\$5$ and $\$1000$. The sample period is 1964 to 2010.

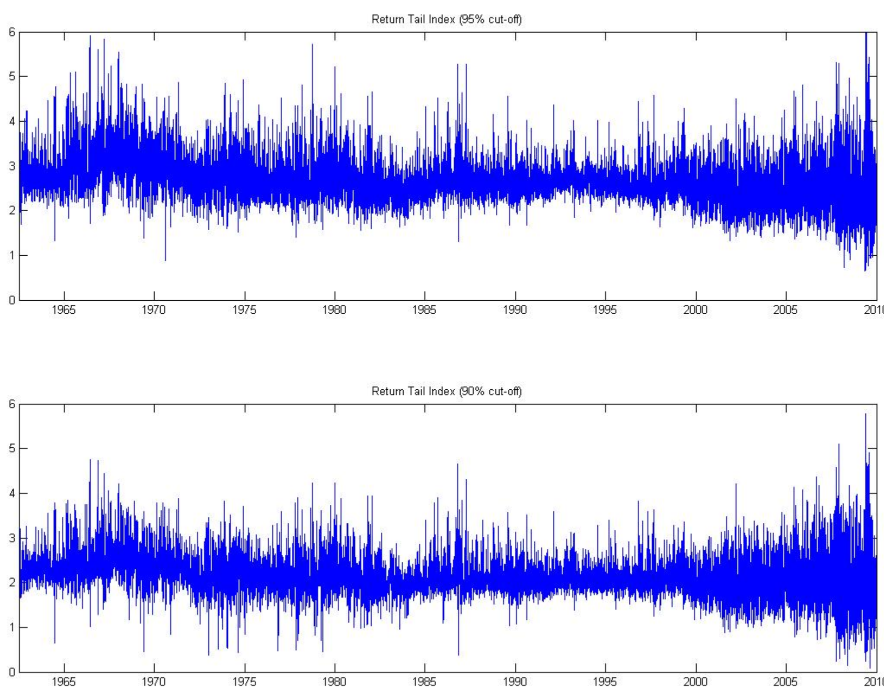

Figure 3: Tail Risk and Volatility: Daily Data

The figure shows the tail index estimator of Hill (1975), applied to stock returns. The tail index is estimated from equation (2) in the cross-section of stocks every day to obtain a daily tail risk index for the market. The data comprise NYSE, AMEX and NASDAQ stocks with prices between $5 and $1000. Also shown is the volatility measure VXO, which measures the implied volatility of an-at-the- money option. The tail index is in green, while the volatility VXO is blue. The sample period is 1986 to 2010.

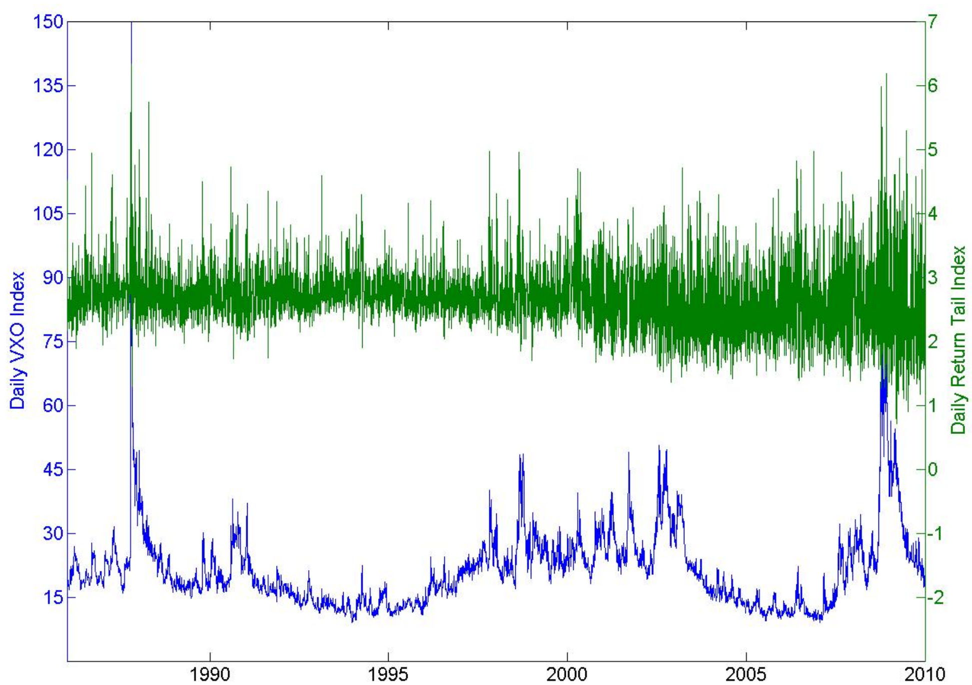

Figure 4: Tail Risk and Volatility: Monthly Data

The figure shows the monthly average of the tail risk estimator of Hill (1975), applied to stock returns. The tail index is estimated from equation (2) in the cross-section of stocks every day to obtain a daily tail risk index for the market. This daily index is then averaged each month to obtain a monthly tail index. The data comprise NYSE, AMEX and NASDAQ stocks with prices between $\$5$ and $\$1000$. Also shown is the monthly average of the volatility measure VXO, which measures implied volatility of an at-the-money option. The tail index is in green, while the volatility VXO is blue. The sample period is 1986 to 2010.

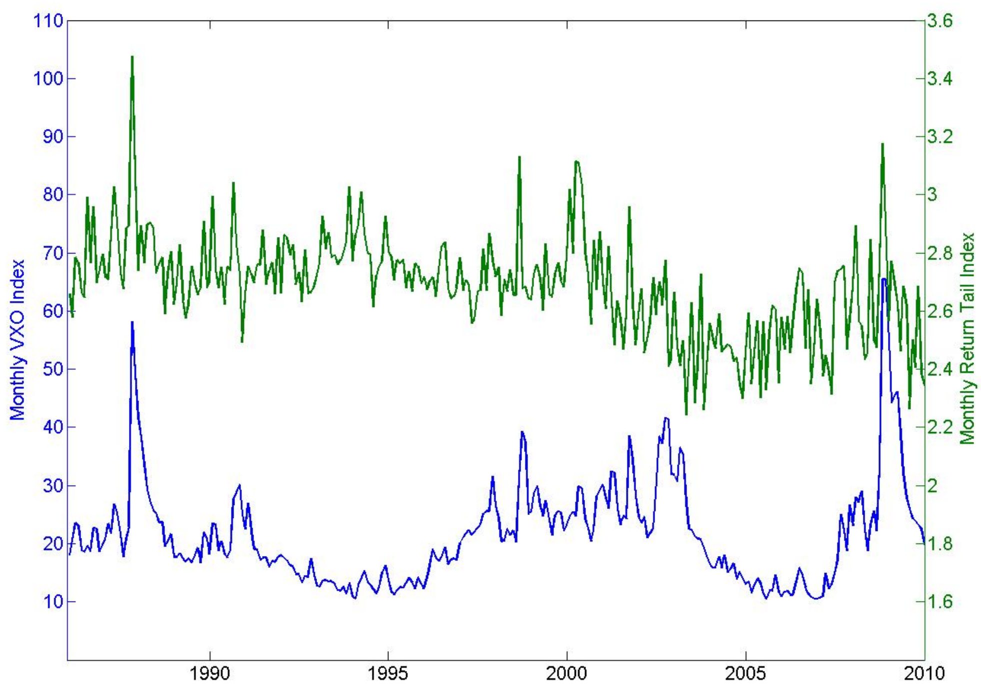

Table 1: Performance of Portfolios based on Return Tail Risk and Liquidity

The table presents the average returns of portfolios sorted on tail risk in returns and liquidity. St. Dev. denotes the standard deviation. The letters 'RTI' and 'LIQ' denote portfolios sorted on sensitivity to the tail index for returns and the liquidity measure of Amihud (2002), respectively, as in equation (5). For example, the portfolio 'LIQ2' corresponds to the returns on firms that are in the second (2) most sensitive quintile to liquidity risk. Portfolio returns are annualized and in percentages, so that 1 represents 1%). The data comprise firms with prices between $5 and $1000, and include common stocks listed on NYSE, AMEX and NASDAQ during the sample period. The time period is 1964 through 2010.

Panel A: Portfolios Sorted on β^R, Sensitivity to Return Tail Index		
	Mean	St. Dev
RTI1	13.48	19.89
RTI2	11.51	16.15
RTI3	11.02	15.07
RTI4	8.94	15.57
RTI5	8.61	20.32
RTI1-RTI5	[**4.87**]	[**3.25**]
Panel B: Portfolios Sorted on β^{liq}, Sensitivity to Liquidity		
LIQ1	11.68	20.29
LIQ2	12.22	16.06
LIQ3	9.99	14.89
LIQ4	10.99	15.50
LIQ5	10.16	21.07
LIQ1-LIQ5	1.52	[0.95]

Table 2: Properties of Return Tail Risk Factor, 'TR'

The table presents summary statistics of the tail risk factor, in terms of average returns and correlation with other factors. t-statistics are in square brackets. The letters 'TR' ('LIQ') denote the return tail risk (liquidity) factor, which is computed as the difference in returns between stocks with highest and lowest sensitivity to the tail index (liquidity). Our measure of liquidity is that of Amihud (2002). We sort all firms in June each year according to their respective sensitivity to tail risk, as estimated in equation (5). We then form 5 quintile portfolios and compute monthly returns over the subsequent year. The return difference between the highest (5) and lowest (1) sensitivity portfolios represents the tail risk factor TR. All factors are annualized and in percentage points, so that 1 represents 1%). Data comprise firms with prices between \$5 and \$1000, and include firms listed on NYSE, AMEX and NASDAQ during the sample period. The time period is 1964 through 2010.

Panel A: Average Returns

	MKT	SMB	HML	LIQ	TR
Mean	5.15	3.49	4.61	1.52	4.87
	[2.21]	[2.14]	[3.05]	[0.95]	[3.25]

Panel B: Correlations

	MKT	SMB	HML	LIQ	TR
MKT	1	0.3088	-0.3066	-0.0501	-0.0322
SMB		1	-0.2348	-0.0559	0.1309
HML			1	0.1765	0.0023
LIQ				1	0.0671
TR					1

Figure 5: The Return Tail Risk Factor

The figure shows the tail risk factor TR, which is computed as the difference in returns between stocks with highest and lowest sensitivity to the daily tail index. We sort all firms in June each year according to their respective sensitivity to tail risk as described in equation (5). We then form 5 quintile portfolios and compute monthly returns over the subsequent year. The return difference between the highest (5) and lowest (1) sensitivity portfolios represents the tail risk factor. The data comprise NYSE, AMEX and NASDAQ stocks with prices between \$5 and \$1000. Shaded areas denote NBER recessions. The sample period is 1964 to 2009.

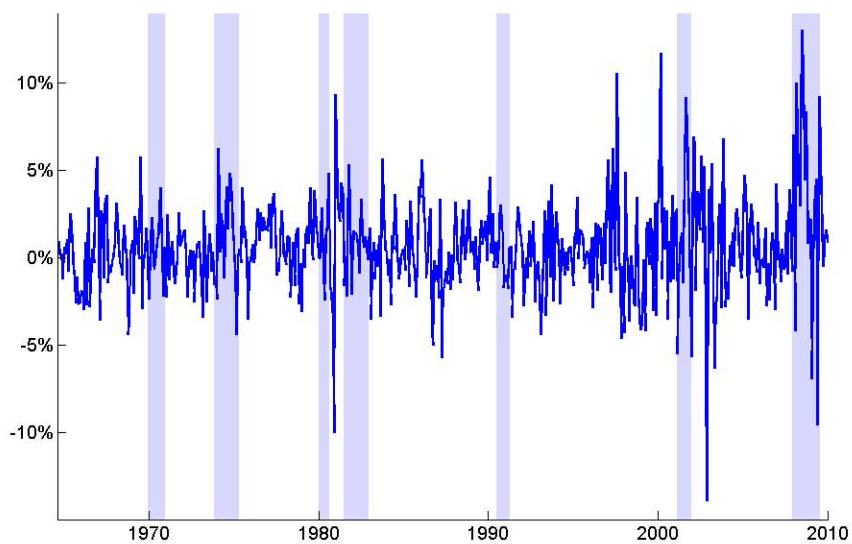

Figure 6: The Exposure of Financial Firms to Tail Risk

The figure presents the proportion of financial firms and other characteristics of the stocks in our Tail Risk portfolios. We sort all firms in June each year according to their respective sensitivity to tail risk as in equation (5). We then form 5 quintile portfolios and compute monthly returns over the subsequent year. The letters 'TI' in the bottom bar denote portfolios sorted on sensitivity to the tail index. For example, the portfolio 'TI2' corresponds to the returns on firms that are in the second (2) most sensitive quintile to tail risk. All portfolios are value weighted. Financial firms are those with SIC code 6000. The proportion of financial firms is in percentage points. Leverage is calculated as the ratio of total debt to total assets. Book-to-Market and leverage are computed using the value in Compustat, as of December 31 of year t-1, for the portfolios that are formed in June of year t. Book-to-market and leverage are winsorized at 1% and 99%. The data comprise NYSE, AMEX and NASDAQ stocks with prices between \$5 and \$1000. The sample period is 1964 to 2009.

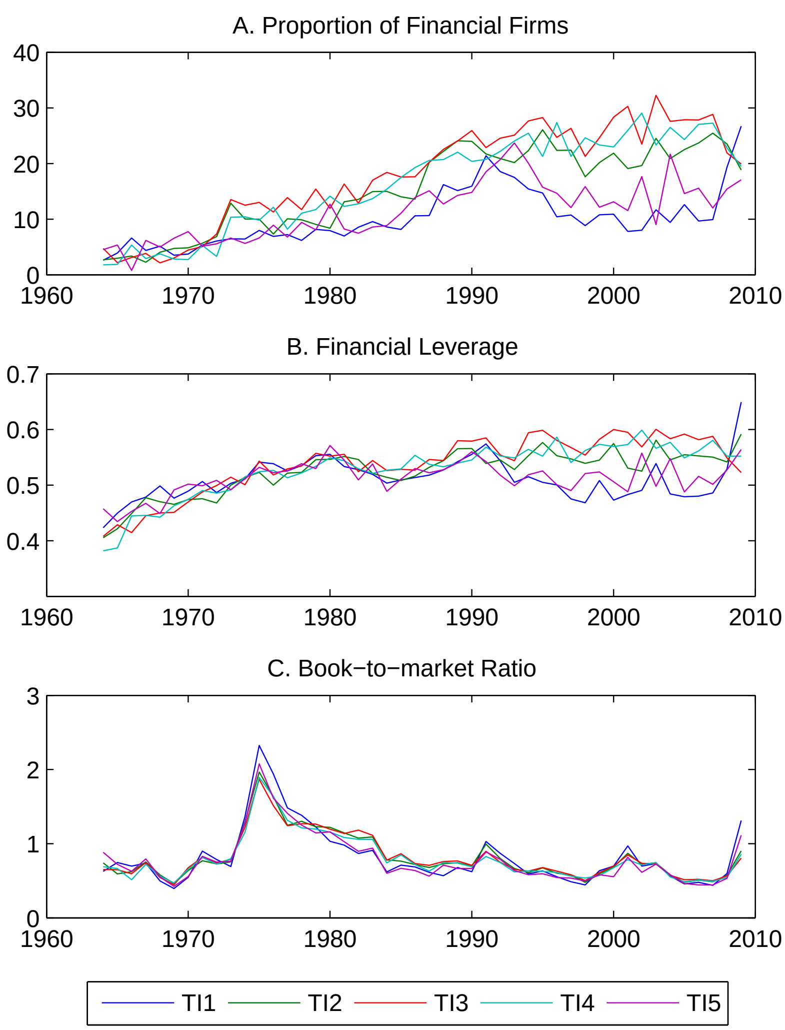

Table 3: Characteristics of Portfolios for Return Tail Risk and Liquidity

The table presents characteristics of the firms in our Return Tail Risk and Liquidity portfolios. We sort all firms in June each year according to their respective sensitivity to return tail risk as in equation (5). We then form 5 quintile portfolios and compute monthly returns over the subsequent year. The letters 'TI' and LIQ denote portfolios sorted on sensitivity to the return tail index and liquidity, respectively. For example, the portfolio 'TI2' corresponds to the returns on firms that are in the second (2) most sensitive quintile to tail risk. Financial firms are those with SIC code 6000. The proportion of financial firms is in percentage points. Leverage is calculated as the ratio of total debt to total assets. Book-to-Market and leverage are computed using the value in Compustat, as of December 31 of year t-1, for the portfolios that are formed in June of year t. Book-to-market and leverage are winsorized at 1% and 99%. Standard deviations are in parentheses. All portfolios are value weighted. The time period comprises 1964 through 2009.

Panel A: Tail Index Portfolios

	TI1	TI2	TI3	TI4	TI5
% Financial Firms	10.06	15.28	17.93	16.32	11.47
	(5.09)	(7.50)	(8.90)	(8.45)	(5.21)
Leverage	0.5103	0.5242	0.5368	0.5276	0.5126
	(0.0370)	(0.0407)	(0.0516)	(0.0485)	(0.0301)
Book-to-Market	0.82	0.82	0.81	0.80	0.79
	(0.39)	(0.32)	(0.30)	(0.30)	(0.33)

Panel B: Liquidity Portfolios

	LIQ1	LIQ2	LIQ3	LIQ4	LIQ5
% Financial Firms	11.01	15.74	17.48	15.84	11.02
	(6.10)	(8.14)	(9.09)	(8.37)	(5.27)
Leverage	0.5148	0.5223	0.5334	0.5277	0.5135
	(0.0384)	(0.0453)	(0.0526)	(0.0442)	(0.0355)
Book-to-Market	0.81	0.82	0.82	0.79	0.78
	(0.37)	(0.31)	(0.30)	(0.29)	(0.36)

Figure 7: Tail Risk for Financial Firms

The figure's upper panel displays the return tail index, computed for all firms, and for only financial firms. The tail index is estimated for daily stock data using the method of Hill (1975), from equation (2), using the cross section of returns each day. Then, we average it across all stocks to obtain a market tail index, reported in the figure's upper panel. The lower panel presents the Dow Jones Industrial Average (DJIA), both levels and returns. The data comprise NYSE, AMEX and NASDAQ stocks with prices between $5 and $1000. The sample period is 1964 to 2010.

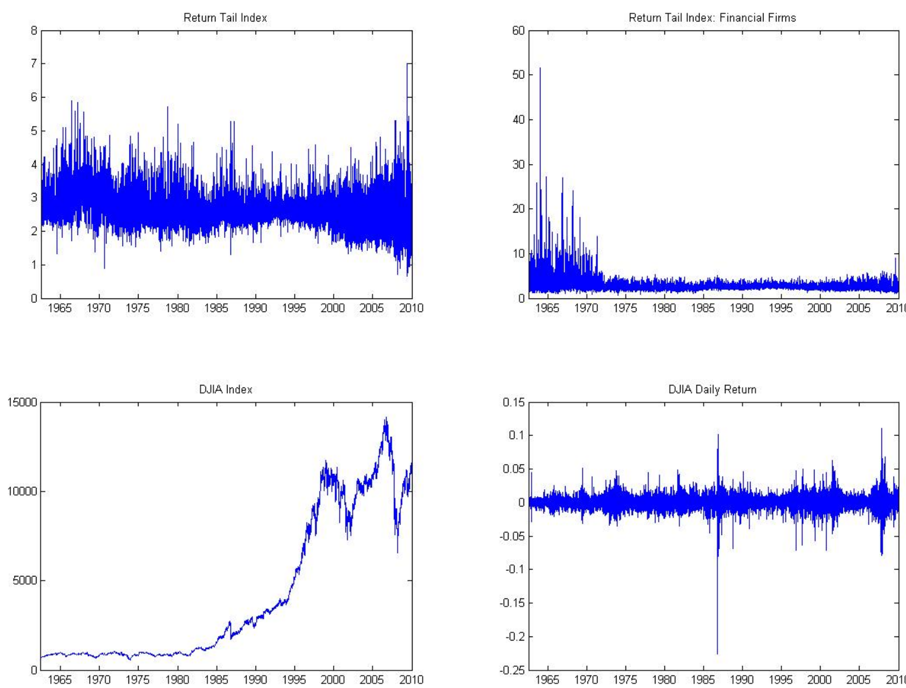

Table 4: Properties of Return Tail Index for Financial Firms

The table presents summary statistics of the return tail index, computed both for all firms and for financial firms only. The tail index is estimated from the cross section of daily returns using the method of Hill (1975), as in equation (2). All tail indices refer to the *left tail*, unless otherwise specified. The market tail index is computed in two steps. First we compute the tail index for each stock from the cross section of returns. Then we average the tail index across all stocks to obtain a market return tail index, reported in the Table. P-values are presented in parentheses. Min, Max and St. Dev. denote minimum, maximum and standard deviation, respectively. RTI and RTI(Fin) denote the return tail index for all companies and for financial companies only, respectively. DJIA is the level of the Dow Jones Industrial Average, and DJIA(Ret) is the daily return on the Dow Jones Industrial Average. Data comprise firms with prices between $5 and $1000, and include firms listed on NYSE, AMEX and NASDAQ during the sample period, January 1964 through December 2010.

Panel A: Summary Statistics

Variable	Mean	St. Dev.	Min	Max
RTI	2.6823	0.5281	0.6467	7.01456
RTI(Fin)	2.8553	1.3897	0.6245	51.5082
DJIA	4164	4090	578	14165
DJIA(Ret)	0.0003	0.0103	-0.2261	0.1108

Panel B: Correlations

	RTI	RTI(Fin)	DJIA	DJIA(Ret)
RTI	1	0.4002	-0.2352	-0.6000
		$(< .0001)$	$(< .0001)$	$(< .0001)$
RTI(Fin)		1	-0.1385	-0.2061
			$(< .0001)$	$(< .0001)$
DJIA			1	0.0060
				(0.5097)
DJIA(Ret)				1

Table 5: Stationarity and Unit Root Tests

The table presents the unit root test of Phillips and Perron (1988), and the KPSS stationarity test of Kwiatkowski et al. (1992). St. Dev is the standard deviation. Month-end TI denotes the tail index calculated at the end of the month. Avg. TI denotes the average tail index each month. TR denotes the tail risk factor. Yield denotes the spread between BAA and AAA bonds, available from the Federal Reserve Bank of St. Louis. Variables are evaluated at the one month frequency. P-values are in parentheses. The time period comprises 1964 through 2009.

	MKT	*Yield*	*TR*	*Month-end TI*	*Avg. TI*
Mean	0.0900	0.0105	0.0047	-2.5913	-2.6919
St. Dev.	0.1588	0.0014	0.0276	0.4735	0.2420
H_0: unit root	-20.50	-1.07	-20.21	-2.84	-0.86
	(0.001)	(0.259)	(0.001)	(0.005)	(0.338)
H_0: stationarity	0.12	3.50	0.15	0.27	1.64
	(0.100)	(0.010)	(0.048)	(0.010)	(0.010)

Table 6: Causality Tests

The table presents causality tests based on the approach of Granger (1969). The framework is a 2-variable vector autoregression (VAR). The null hypothesis is that all coefficients are zero, which is assessed by an F-test. The symbols TR and MKT denote the tail risk factor and market return, respectively. Yield denotes the spread between BAA and AAA bonds, available from the Federal Reserve Bank of St. Louis. The tail index is evaluated at the month's end. AIC and BIC are the Akaike and Bayesian information criteria. All dependent variables are annualized. The symbols ***, **, and * denote p-values smaller than 0.01, 0.05, and 0.10, respectively. The time period comprises 1964 through 2009.

Panel A: Choice of Optimal Lag in Bivariate Vector Autoregression

	Model 1 TR vs Yield		Model 2 TR vs MKT		Model 3 Tail Index vs Yield		Model 4 Tail Index vs MKT	
Lags	AIC	BIC	AIC	BIC	AIC	BIC	AIC	BIC
1	-6075.59	-6049.90	**-7129.25**	**-7103.57**	-3051.74	-3026.06	-4101.41	**-4075.73**
2	-6126.04	**-6083.23**	-7123.85	-7081.05	-3111.37	**-3068.56**	-4111.34	-4068.54
3	-6142.19	-6082.27	-7120.08	-7060.15	-3117.88	-3057.95	-4108.20	-4048.28
4	-6137.17	-6060.12	-7112.71	-7035.66	-3117.13	-3040.08	-4106.25	-4029.20
5	-6142.05	-6047.88	-7114.19	-7020.02	-3123.22	-3029.05	-4116.80	-4022.63
6	**-6157.90**	-6046.61	-7110.22	-6998.92	-3134.50	-3023.21	**-4116.80**	-4005.51
7	-6153.01	-6024.59	-7103.13	-6974.72	-3130.73	-3002.32	-4110.59	-3982.18
8	-6154.61	-6009.08	-7098.34	-6952.81	-3135.31	-2989.78	-4106.94	-3961.41
9	-6150.43	-5987.78	-7092.98	-6930.33	-3134.19	-2971.54	-4101.88	-3939.22
10	-6146.08	-5966.30	-7089.22	-6909.44	-3135.35	-2955.58	-4102.89	-3923.11
11	-6149.27	-5952.38	-7088.50	-6891.61	-3133.68	-2936.78	-4102.03	-3905.13
12	-6145.03	-5931.01	-7082.72	-6868.70	**-3137.82**	-2923.80	-4103.86	-3889.84

Panel B: F-statistic from Granger Causality test (lag determined by BIC)

	Model 1: TR vs Yield		*Model 2: TR vs MKT*		*Model 3: Tail Index vs Yield*		*Model 4: Tail Index vs MKT*	
	Variable 1 TR	Variable 2 Yield	Variable 1 TR	Variable 2 MKT	Variable 1 Tail Index	Variable 2 Yield	Variable 1 Tail Index	Variable 2 MKT
Variable 1	4.40**	5.73***	6.34**	0.20	7.66***	1.95	3.01*	3.35*
Variable 2	1.65	4110.61***	4.20**	4.61**	1.46	3973.72***	3.34*	5.18**

Panel C: F-statistic from Granger Causality test (lag determined by AIC)

	Model 1: TR vs Yield		*Model 2: TR vs MKT*		*Model 3: Tail Index vs Yield*		*Model 4: Tail Index vs MKT*	
	Variable 1 TR	Variable 2 Yield	Variable 1 TR	Variable 2 MKT	Variable 1 Tail Index	Variable 2 Yield	Variable 1 Tail Index	Variable 2 MKT
Variable 1	2.16**	6.15***	6.34**	0.20	4.86***	0.72	6.47***	1.35
Variable 2	2.36**	1339.12***	4.20**	4.61**	1.50	692.76***	2.25**	1.90*

Table 7: Predictability Tests

The table presents tests of predictability for the market return (MKT) and yield spread (Yield), using the tail index and tail risk factor TR. In Panel A, the regression tests are of the form $MKT_t = \alpha + \beta^{TR} \cdot TR_{t-1} + \varepsilon_t$ and $Yield_t = \alpha + \beta^{TR} \cdot TR_{t-1} + \varepsilon_t$. Panel B presents the same estimation except that the tail index replaces TR on the right hand side, and the coefficient is now β^{TI}. All dependent variables are annualized so that coefficients can be compared directly. Yield denotes the spread between BAA and AAA bonds, available from the Federal Reserve Bank of St. Louis. The tail index is evaluated at the month's end. We consider horizons of 1 month, 1 year, 3 years, and 5 years. Standard errors are in square brackets, and computed using the method of Hodrick (1992). The time period comprises 1964 through 2009.

Panel A: Predictability using Tail Risk Factor TR

	Dependent Variable: MKT_t				Dependent Variable: $Yield_t$			
	1 Month	1 Year	3 Year	5 Year	1 Month	1 Year	3 Year	5 Year
α	0.0904	0.0900	0.0937	0.0954	0.0104	0.0104	0.0104	0.0105
	[0.0236]	[0.0238]	[0.0242]	[0.0244]	[0.0002]	[0.0002]	[0.0002]	[0.0002]
β^{TR}	-0.0893	-0.7188	0.0486	0.1002	0.0113	0.0271	0.0079	0.0064
	[1.0322]	[0.2974]	[0.1109]	[0.0892]	[0.0085]	[0.0025]	[0.0009]	[0.0007]

Panel B: Predictability Using Market Tail Index

	Dependent Variable: MKT_t				Dependent Variable: $Yield_t$			
	1 Month	1 Year	3 Years	5 Years	1 Month	1 Year	3 Years	5 Years
α	-0.1286	0.0943	0.1267	0.1292	0.0142	0.0123	0.0109	0.0105
	[0.1395]	[0.0558]	[0.0543]	[0.0501]	[0.0013]	[0.0004]	[0.0005]	[0.0005]
β^{TI}	-0.0820	0.0029	0.0123	0.0125	0.0014	0.0007	0.0002	0.0000
	[0.0528]	[0.0200]	[0.0189]	[0.0181]	[0.0005]	[0.0001]	[0.0002]	[0.0002]

Table 8: The Price of Exposure to Tail Risk

The table presents the estimated risk premia, which measure the return per unit of exposure to each risk factor. More details are in the Appendix. The letters 'TR' ('LIQ') denote the return tail risk (liquidity) factor, which is computed as the difference in returns between stocks with highest and lowest sensitivity to the tail index (liquidity). We sort all firms in June each year according to their respective sensitivity to tail risk as in equation (5). We then form 5 quintile portfolios and compute monthly returns over the subsequent year. The return difference between the highest (RTI5) and lowest (RTI1) sensitivity portfolios represents the tail risk factor. FF3 denotes the Fama-French 3-factor model. The test statistics are the 5x5 book to market portfolios, available from the website of Kenneth French. Estimation is performed by GMM of Hansen (1982). Robust t-statistics are in square brackets. The data comprise common stocks on NASDAQ, NYSE and AMEX with at least 120 trading days in the relevant year. The time period is years 1964 through 2009.

Model:	*CAPM*	*CAPM & LIQ*	*CAPM & TR*	*CAPM & LIQ, TR*	*FF3*	*FF3 & LIQ, TR*
Estimated Risk Premia						
MKT	0.0037	0.0030	**0.0058**	**0.0063**	**0.0046**	**0.0060**
	[1.89]	[1.57]	**[2.38]**	**[2.46]**	**[2.14]**	**[2.10]**
SMB					0.0014	0.0041
					[1.00]	[2.00]
HML					**0.0046**	**0.0043**
					[3.29]	**[1.96]**
LIQ		0.0015		-0.0029		0.0047
		[0.4156]		[-0.56]		[-0.78]
TR			**0.0190**	**0.0213**		**0.0234**
			[3.64]	**[3.67]**		**[3.54]**

Table 9: Asset Pricing Tests

The table presents the results of asset pricing tests on our sample. Estimation is performed using GMM. Robust t-statistics are in square brackets, and p-values are in parentheses. The J-test is the over-identifying restriction test of Hansen (1982). HJ-distance refers to the distance metric of Hansen and Jagannathan (1997). Large p-values for the J-statistic and HJ distance indicate that the particular model fits well. The delta-J test of Newey and West (1987) assesses whether the inclusion of HML and SMB improves model fit. A small p-value for the delta-J test indicates that additional factors improve model fit. The letters 'TR' ('LIQ') denote the return tail risk (liquidity) factor, which is computed as the difference in returns between stocks with highest and lowest sensitivity to the tail index (liquidity). We sort all firms in June each year according to their respective sensitivity to tail risk as in equation (5). We then form 5 quintile portfolios and compute monthly returns over the subsequent year. The return difference between the highest (RTI5) and lowest (RTI1) sensitivity portfolios represents the tail risk factor. All portfolios are value weighted. FF3 denotes the Fama-French 3-factor model. The data comprise common stocks on NASDAQ, NYSE and AMEX with at least 120 trading days in the relevant year. The time period is 1964 through 2009.

Model:	*CAPM*	*CAPM & LIQ*	*CAPM & TR*	*CAPM & LIQ, TR*	*FF3*	*FF3 & LIQ, TR*
J-Statistic	46.45	46.89	31.78	29.53	36.82	20.13
	(0.00)	(0.00)	(0.08)	(0.10)	(0.02)	(0.39)
HJ Distance	0.36	0.36	0.31	0.31	0.31	0.25
	(0.00)	(0.00)	(0.07)	(0.05)	(0.00)	(0.41)
Delta-J	9.63	13.27	9.58	9.39		
	(0.01)	(0.00)	(0.01)	(0.01)		

Table 10: Robustness to Value-Weighted Liquidity

The table presents estimated premia and results of asset pricing tests on our sample, where we calculate liquidity portfolios that are value-weighted. Estimation is performed using GMM. Robust t-statistics are in square brackets, and p-values are in parentheses. The J-test is the over-identifying restriction test of Hansen (1982). HJ-distance refers to the distance metric of Hansen and Jagannathan (1997). Large p-values for the J-statistic and HJ distance indicate that the particular model fits well. The delta-J test of Newey and West (1987) assesses whether the inclusion of HML and SMB improves model fit. A small p-value for the delta-J test indicates that additional factors improve model fit. The letters 'TR' ('VWLIQ') denote the return tail risk (liquidity) factor, which is computed as the difference in returns between stocks with highest and lowest sensitivity to the tail index (liquidity). We sort all firms in June each year according to their respective sensitivity to tail risk as in equation (5). We then form 5 quintile portfolios and compute monthly returns over the subsequent year. The return difference between the highest (RTI5) and lowest (RTI1) sensitivity portfolios represents the tail risk factor. All portfolios are value weighted. FF3 denotes the Fama-French 3-factor model. The data comprise common stocks on NASDAQ, NYSE and AMEX with at least 120 trading days in the relevant year.The time period is 1964 through 2009.

Model:	*CAPM*	*CAPM & VWLIQ*	*CAPM & TR*	*CAPM & VWLIQ, TR*	*FF3*	*FF3 & VWLIQ, TR*
Panel A: Estimated Risk Premia						
MKT	0.0035	0.0040	**0.0055**	**0.0057**	**0.0044**	0.0051
	[1.78]	[1.99]	[**2.29**]	[**2.33**]	[**2.03**]	[1.57]
SMB					0.0013	0.0030
					[0.89]	[1.51]
HML					**0.0047**	**0.0062**
					[**3.33**]	[**2.79**]
VWLIQ		-0.0019		-0.0015		0.0048
		[-0.50]		[-0.34]		[0.88]
TR			**0.0133**	**0.0133**		**0.0236**
			[**2.78**]	[**2.76**]		[**3.16**]
Panel B: Asset Pricing Tests						
J-Statistic	45.96	44.55	35.69	35.54	36.37	18.35
	(0.00)	(0.00)	(0.03)	(0.02)	(0.02)	(0.50)
HJ Distance	0.36	0.36	0.32	0.31	0.31	0.25
	(0.00)	(0.00)	(0.11)	(0.13)	(0.00)	(0.52)
Delta-J	9.59	8.10	14.29	17.18		
	(0.01)	(0.02)	(0.00)	(0.00)		

Table 11: Robustness to Liquidity and Momentum

The table presents estimated premia and results of asset pricing tests on our sample, where we account for an alternative liquidity factor and momentum effects. Estimation is performed using GMM. Robust t-statistics are in square brackets, and p-values are in parentheses. The J-test is the over-identifying restriction test of Hansen (1982). HJ-distance refers to the distance metric of Hansen and Jagannathan (1997). Large p-values for the J-statistic and HJ distance indicate that the particular model fits well. The delta-J test of Newey and West (1987) assesses whether the inclusion of HML and SMB improves model fit. A small p-value for the delta-J test indicates that additional factors improve model fit. The letters 'TR' denote the return tail risk factor, which is computed as the difference in returns between stocks with highest and lowest sensitivity to the tail index. We sort all firms in June each year according to their respective sensitivity to tail risk as in equation (5). We then form 5 quintile portfolios and compute monthly returns over the subsequent year. The return difference between the highest (RTI5) and lowest (RTI1) sensitivity portfolios represents the tail risk factor. All portfolios are value weighted. FF3 denotes the Fama-French 3-factor model. PSLIQ is the liquidity factor of Pastor and Stambaugh (2003), and MOM is the momentum factor of Carhart (1997). The data comprise common stocks on NASDAQ, NYSE and AMEX with at least 120 trading days in the relevant year. The time period is 1964 through 2009.

Model:	*CAPM & PSLIQ, MOM*	*CAPM & PSLIQ & MOM, TR*	*FF3*	*FF3 & PSLIQ & MOM, TR*
Panel A: Estimated Risk Premia				
MKT	**0.0059**	0.0053	**0.0047**	0.0049
	[**2.05**]	[1.80]	[**2.03**]	[1.59]
SMB			0.0010	0.0015
			[0.68]	[0.71]
HML			**0.0046**	**0.0048**
			[**3.18**]	[**2.54**]
PSLIQ	**0.0278**	**0.0253**		0.0122
	[**3.24**]	[[**2.90**]]		[1.41]
MOM	0.0043	0.0041		0.0129
	[0.57]	[0.48]		[1.49]
TR		**0.0118**		**0.0132**
		[**2.29**]		[**2.33**]
Panel B: Asset Pricing Tests				
J-Statistic	24.93	19.45	32.79	18.71
	(0.25)	(0.49)	(0.05)	(0.41)
HJ Distance	0.34	0.30	0.31	0.22
	(0.03)	(0.20)	(0.01)	(0.64)
Delta-J	4.59	0.74		
	(0.10)	(0.69)		

Table 12: Robustness to Downside Risk

The table presents estimated premia and results of asset pricing tests on our sample, where we account for downside risk. As in Ang et al. (2006a) equation (13), we consider both a market factor and a 'downside' factor that equals the minimum of the current market return and its historical average. That is, we estimate linear factor models of the form $r^e_{it} = \beta^0 + \beta^M MKT_t + \beta^D MKT^-_t + \beta^S SMB_t + \beta^H HML_t + \beta^L LIQ_t + \beta^T TR_t + \varepsilon_{it}$ where MKT^-_t is the minimum of the market return and its historical average computed up to period t. Further, r^e_i and MKT represent excess returns on individual stocks and the market portfolio, SMB and HML are the Fama-French factors, and $Illiq$ and TI are the Amihud liquidity measure and tail index of cross-sectional daily returns described above.Estimation is performed using GMM. Robust t-statistics are in square brackets, and p-values are in parentheses. The J-test is the over-identifying restriction test of Hansen (1982). HJ-distance refers to the distance metric of Hansen and Jagannathan (1997). Large p-values for the J-statistic and HJ distance indicate that the particular model fits well. The delta-J test of Newey and West (1987) assesses whether the inclusion of HML and SMB improves model fit. A small p-value for the delta-J test indicates that additional factors improve model fit. The tail risk factor is the return difference between a high risk portfolio and low risk portfolio. We sort all firms in June each year according to their respective sensitivity to tail risk as in equation (5). We then form 5 quintile portfolios and compute monthly returns over the subsequent year. The return difference between the highest (RTI5) and lowest (RTI1) sensitivity portfolios represents the tail risk factor. All portfolios are value weighted. The time period comprises 1964 through 2009.

Model:	*CAPM &Down*	*CAPM & LIQ &TR, Down*	*FF3 &Down*	*FF3 &LIQ & TR, Down*
Panel A: Estimated Risk Premia				
MKT	0.0030	0.0045	**0.0045**	0.0032
	[1.30]	[1.71]	[**2.23**]	[0.98]
SMB			0.0014	0.0022
			[0.97]	[1.06]
HML			**0.0046**	**0.0053**
			[**3.31**]	[**2.43**]
LIQ		0.0030		0.0030
		[0.70]		[0.56]
TR		**0.0148**		**0.0232**
		[**3.12**]		[**3.35**]
Down	0.0032	0.0029	0.0018	0.0028
	[1.15]	[0.88]	[0.61]	[0.66]
Panel B: Asset Pricing Tests				
J-Statistic	44.74	32.30	36.29	18.99
	(0.00)	(0.04)	(0.01)	(0.39)
HJ Distance	0.36	0.31	0.30	0.25
	(0.00)	(0.06)	(0.00)	(0.52)
Delta-J	8.45	13.30		
	(0.01)	(0.00)		

Table 13: Return Properties of Tail Risk Factor TR and Volatility factor FVIX

The table presents summary statistics of TR and the FVIX volatility risk factor of Ang et al. (2006b), in terms of average returns and correlation with other factors. Standard deviations are denoted St. Dev. The letters 'TR' ('LIQ') denote the return tail risk (liquidity) factor, which is computed as the difference in returns between stocks with highest and lowest sensitivity to the tail index (liquidity). We sort all firms in June each year according to their respective sensitivity to tail risk as in equation (5). We then form 5 quintile portfolios and compute monthly returns over the subsequent year. The return difference between the highest (RTI5) and lowest (RTI1) sensitivity portfolios represents the tail risk factor. Data comprise firms with prices between \$5 and \$1000, and include firms listed on NYSE, AMEX and NASDAQ during the sample period. The time period is February 1986 to December 2009.

Panel A: Average Returns

	MKT	SMB	HML	LIQ	TR	FVIX
Mean	6.32	0.93	3.64	1.40	6.05	0.54
St. Dev	16.07	11.72	11.08	12.17	11.05	2.13

Panel B: Correlations

	MKT	SMB	HML	LIQ	TR	FVIX
MKT	1	0.2101	-0.3136	-0.0790	-0.1046	-0.6630
SMB		1	-0.3483	-0.1470	0.0978	-0.1376
HML			1	0.2935	-0.0022	0.1988
LIQ				1	0.2883	0.0014
TR					1	0.0416
FVIX						1

Table 14: Robustness to Volatility Risk

The table presents estimated premia and results of asset pricing tests on our sample, where we account for the volatility factor FVIX of Ang et al. (2006b). Estimation is performed using GMM. Robust t-statistics are in square brackets, and p-values are in parentheses. The J-test is the over-identifying restriction test of Hansen (1982). HJ-distance refers to the distance metric of Hansen and Jagannathan (1997). Large p-values for the J-statistic and HJ distance indicate that the particular model fits well. The delta-J test of Newey and West (1987) assesses whether the inclusion of HML and SMB improves model fit. A small p-value for the delta-J test indicates that additional factors improve model fit. The tail risk factor is the return difference between a high risk portfolio and low risk portfolio. We sort all firms in June each year according to their respective sensitivity to tail risk as in equation (5). We then form 5 quintile portfolios and compute monthly returns over the subsequent year. The return difference between the highest (RTI5) and lowest (RTI1) sensitivity portfolios represents the tail risk factor. All portfolios are value weighted. The time period comprises 1986 through 2009.

Model:	*CAPM & FVIX*	*CAPM &LIQ & TR, FVIX*	*FF3 & FVIX*	*FF3 & LIQ & TR, FVIX*
Panel A: Estimated Risk Premia				
MKT	0.0017	0.0099	0.0018	0.0092
	[0.43]	[1.59]	[0.31]	[1.24]
SMB			**-0.0073**	0.0007
			[**-2.89**]	[0.23]
HML			**0.0081**	0.0032
			[**4.49**]	[1.02]
LIQ		0.0023		0.0030
		[0.47]		[0.53]
TR		**0.0205**		**0.0266**
		[**3.78**]		[**3.93**]
FVIX	-0.0012	-0.0028	-0.0029	-0.0032
	[-0.95]	[-1.46]	[-1.35]	[-1.46]
Panel B: Asset Pricing Tests				
J-Statistic	35.35	25.66	28.36	14.42
	(0.04)	(0.18)	(0.10)	(0.70)
HJ Distance	0.46	0.39	0.44	0.33
	(0.00)	(0.16)	(0.00)	(0.57)
Delta-J	6.99	11.24		
	(0.03)	(0.00)		

Table 15: Robustness to Price Filter

The table presents estimated premia and results of asset pricing tests on our sample, where we filter the data to include only stocks with prices in the range \$5 - \$1000 in the previous year. Estimation is performed using GMM. Robust t-statistics are in square brackets, and p-values are in parentheses. The J-test is the over-identifying restriction test of Hansen (1982). HJ-distance refers to the distance metric of Hansen and Jagannathan (1997). Large p-values for the J-statistic and HJ distance indicate that the particular model fits well. The delta-J test of Newey and West (1987) assesses whether the inclusion of HML and SMB improves model fit. A small p-value for the delta-J test indicates that additional factors improve model fit. The tail risk factor is the return difference between a high risk portfolio and low risk portfolio. We sort all firms in June each year according to their respective sensitivity to tail risk, as in equation (5). We then form 5 quintile portfolios and compute monthly returns over the subsequent year. The return difference between the highest (RTI5) and lowest (RTI1) sensitivity portfolios represents the tail risk factor. All portfolios are value weighted. The time period comprises 1964 through 2009.

Model:	*CAPM*	*CAPM &LIQ*	*CAPM & TR*	*CAPM & LIQ, TR*	*FF3*	*FF3 & LIQ, TR*
Panel A: Estimated Risk Premia						
MKT	0.0035	0.0025	0.0050	0.0046	**0.0044**	0.0046
	[1.78]	[1.24]	[1.90]	[1.71]	[**2.03**]	[1.56]
SMB					0.0013	0.0022
					[0.89]	[1.23]
HML					**0.0047**	**0.0049**
					[**3.33**]	[**2.33**]
LIQ		0.0042		0.0064		0.0060
		[1.08]		[1.24]		[1.04]
TR			**0.0171**	**0.0173**		**0.0193**
			[**2.77**]	[**2.79**]		[**2.78**]
Panel B: Asset Pricing Tests						
J-Statistic	45.96	45.39	30.69	30.46	36.37	22.63
	(0.00)	(0.00)	(0.10)	(0.08)	(0.02)	(0.25)
HJ Distance	0.36	0.36	0.32	0.32	0.31	0.27
	(0.00)	(0.00)	(0.07)	(0.05)	(0.00)	(0.25)
Delta-J	9.59	9.34	7.94	7.83		
	(0.01)	(0.01)	(0.02)	(0.02)		

Figure 8: Comparison of Tail Index for Returns and Liquidity

The figure shows the tail index estimated from the daily cross section of firm liquidity and returns. The estimation is performed using the method of Hill (1975), as in equation (2). The liquidity measure is that of Amihud (2002). The return tail index is in green, the liquidity tail index is in blue. The data comprise NYSE, AMEX and NASDAQ stocks from CRSP, with prices between $\$5$ and $\$1000$. The sample period is 1964 to 2010.

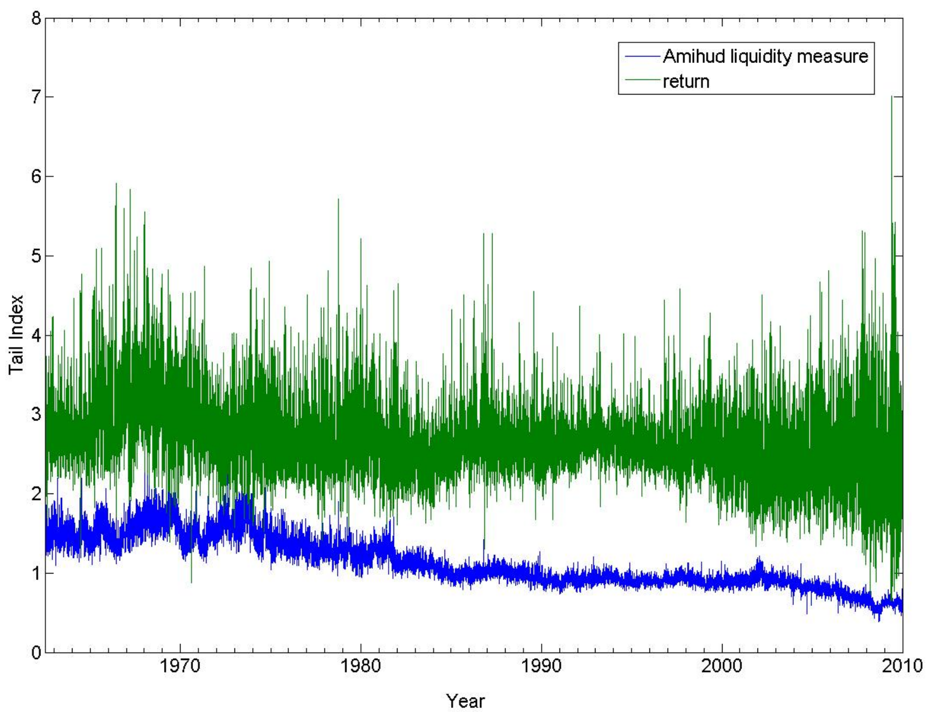

Table 16: Performance of Liquidity Tail Risk Portfolios

The table presents the average returns of portfolios sorted on liquidity tail risk and liquidity levels. Standard deviations are in the column headed 'St. Dev', and t-statistics for significance of the high-low differentials are in square brackets. The letters 'LTI' and LIQ denote portfolios sorted on sensitivity to the liquidity tail index and liquidity measure of Amihud (2002), respectively, as in equation (6). For example, the portfolio 'LTI2' corresponds to the returns on firms that are in the second (2) most sensitive quintile to liquidity tail risk. Returns are annualized and in percentages, so that 1 represents 1%). The data comprise firms with prices between $5 and $1000, and include common stocks listed on NYSE, AMEX and NASDAQ during the sample period. The time period is 1964 through 2010.

Panel A: Portfolios sorted on β^L, Sensitivity to Liquidity Tail Index

	Mean	St. Dev
LTI1	11.17	20.44
LTI2	11.00	16.07
LTI3	10.70	14.99
LTI4	11.05	15.64
LTI5	10.44	19.51
LTI1-LTI5	0.72	
t-value	[0.52]	

Panel B: Portfolios sorted on β^{liq}, Sensitivity to Liquidity Level

	Mean	St. Dev
LIQ1	11.69	20.17
LIQ2	12.02	16.19
LIQ3	10.19	14.79
LIQ4	10.66	15.68
LIQ5	10.67	20.11
LIQ1-LIQ5	1.02	
t-value	[0.68]	

Table 17: Properties of Liquidity Tail Risk Factor, 'LTR'

The table presents summary statistics of the liquidity tail risk factor, in terms of average returns and correlation with other factors. t-statistics are in square brackets. The letters 'LTR' ('LIQ') denote the liquidity tail risk (liquidity level) factor, which is computed as the difference in returns between stocks with highest and lowest sensitivity to the tail index (liquidity). Our measure of liquidity is that of Amihud (2002). We sort all firms in June each year according to their respective sensitivity to liquidity tail risk, as estimated in equation (6). We then form 5 quintile portfolios and compute monthly returns over the subsequent year. The return difference between the highest (LTI5) and lowest (LTI1) sensitivity portfolios represents the tail risk factor LTR. All factors are annualized and in percentage points, so that 1 represents 1%). Data comprise firms with prices between \$5 and \$1000, and include firms listed on NYSE, AMEX and NASDAQ during the sample period. The time period is 1964 through 2010.

Panel A: Average Returns

	MKT	SMB	HML	LIQ	LTR
Mean	5.15	3.49	4.61	1.02	0.72
	[2.21]	[2.14]	[3.05]	[0.68]	[0.52]

Panel B: Correlations

	MKT	SMB	HML	LIQ	LTR
MKT	1	0.3088	-0.3066	0.0190	0.0958
SMB		1	-0.2348	-0.0287	0.1080
HML			1	0.1634	-0.0567
LIQ				1	0.1410
LTR					1

Table 18: Asset Pricing Tests for Liquidity Tail Risk "LTR"

The table presents estimated premia and results of asset pricing tests on our sample. Panel A displays risk premia, which measure the return per unit of exposure to each risk factor. More details are in the Appendix. The letters 'LTR' ('LIQ') denote the liquidity tail risk (liquidity level) factor, which is computed as the difference in returns between stocks with highest and lowest sensitivity to the liquidity tail index (liquidity level). Estimation is performed using GMM. Robust t-statistics are in square brackets. Panel B displays results from three asset pricing tests. The J-test is the over-identifying restriction test of Hansen (1982). HJ-distance refers to the distance metric of Hansen and Jagannathan (1997). Large p-values for the J-statistic and HJ distance indicate that the particular model fits well. The delta-J test of Newey and West (1987) assesses whether the inclusion of HML and SMB improves model fit. Robust p-values are in parentheses. A small p-value for the delta-J test indicates that additional factors improve model fit. We sort all firms in June each year according to their respective sensitivity to tail risk as in equation (6). We then form 5 quintile portfolios and compute monthly returns over the subsequent year. The return difference between the highest (LTI5) and lowest (LTI1) sensitivity portfolios represents the liquidity tail risk factor LTR. All portfolios are value weighted. FF3 denotes the Fama-French 3-factor model. The data comprise common stocks on NASDAQ, NYSE and AMEX with at least 120 trading days in the relevant year. The time period is 1964 through 2010.

Model:	*CAPM*	*CAPM & LIQ*	*CAPM & LTR*	*CAPM & LIQ, LTR*	*FF3*	*FF3 & LIQ, LTR*
Panel A: Estimated Risk Premia						
MKT	0.0037	0.0027	0.0043	0.0035	0.0046	0.0062
	[1.89]	[1.41]	[2.13]	[1.75]	[2.14]	[2.73]
SMB					0.0014	0.0017
					[1.00]	[1.18]
HML					0.0046	0.0040
					[3.29]	[2.68]
LIQ		0.0028		0.0029		-0.0024
		[0.81]		[0.83]		[-0.58]
LTR			0.0044	0.0041		0.0048
			[1.44]	[1.32]		[1.54]
Panel B: Asset Pricing Tests						
J-Statistic	46.45	47.19	44.59	45.46	36.82	31.12
	(0.00)	(0.00)	(0.00)	(0.00)	(0.02)	(0.04)
HJ Distance	0.36	0.36	0.35	0.35	0.31	0.30
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Delta-J	9.63	11.99	10.67	14.34		
	(0.01)	(0.00)	(0.00)	(0.00)		

Figure 9: High Frequency-based Tail Index in Returns

The figure displays the tail index in firm returns, based on intraday data. The computation comprises two steps. First, each day we compute minute-by-minute returns for stocks on NYSE-TAQ, then estimate the tail index for each stock, based on the 5% extreme returns. The estimation is performed using the method of Hill (1975), as in equation (2). Second, we average all firm tail indices to obtain an aggregate market tail index, reported in the figure. The data comprise NYSE, AMEX and NASDAQ stocks with prices between \$5 and \$1000. The sample period is January 1993 to December 2010.

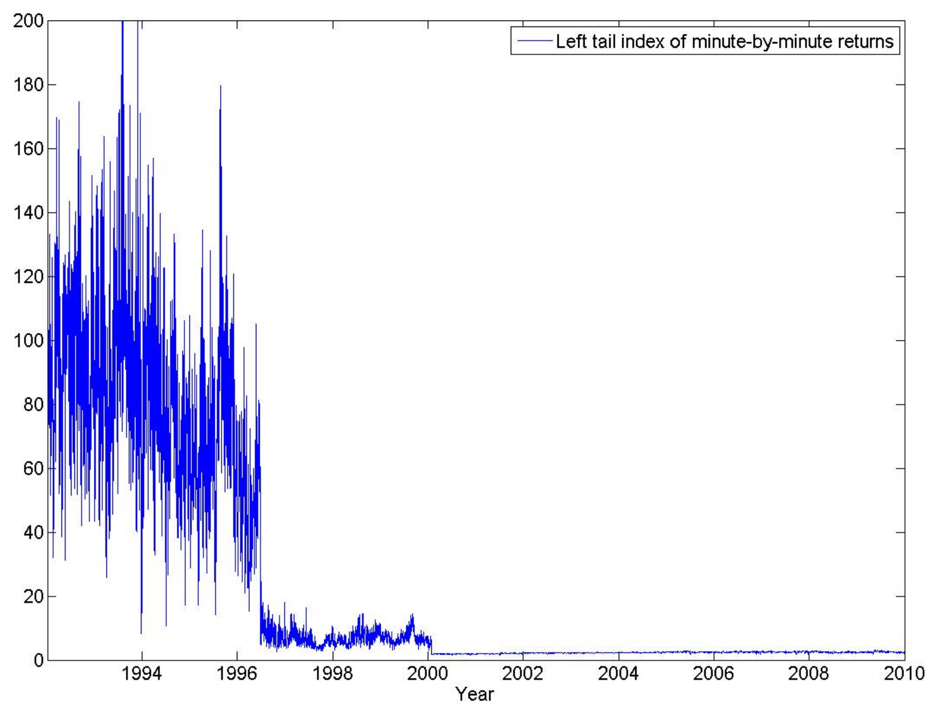

Figure 10: High Frequency-based Tail Index in Liquidity

The figure displays liquidity tail indices computed from intraday data. The intraday-based liquidity tail index is computed in two steps. First we compute the tail index for each stock from minute-by-minute liquidity (effective spread, absolute spread, and relative spread), using NYSE-TAQ stocks. Second, we average the tail index across all stocks to obtain a market liquidity tail index, reported in the figure. The data comprise NYSE, AMEX and NASDAQ stocks with prices between $5 and $1000. The sample period is 1993 to 2010.

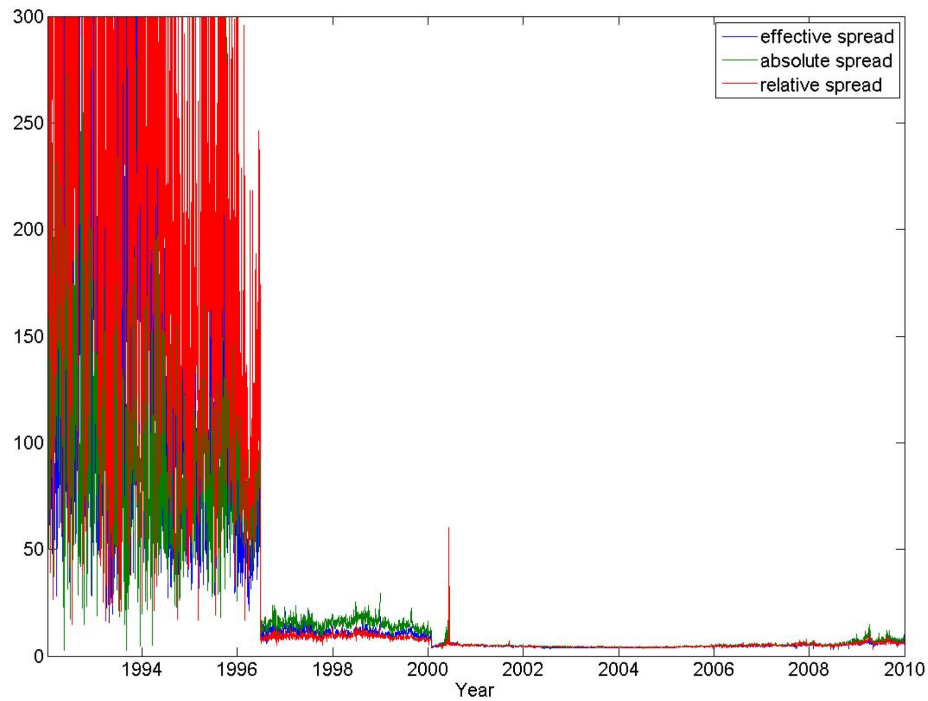

Figure 11: High Frequency-based Tail Index in Returns: 2001-2010

The figure displays the tail index in firm returns, based on intraday data. The computation comprises two steps. First, each day we compute minute-by-minute returns for stocks on NYSE-TAQ, then estimate the tail index for each stock, based on the 5% extreme returns. The estimation is performed using the method of Hill (1975), as in equation (2). Second, we average all firm tail indices to obtain an aggregate market tail index, reported in the figure. The data comprise NYSE, AMEX and NASDAQ stocks with prices between \$5 and \$1000. The sample period is January 2001 to December 2010.

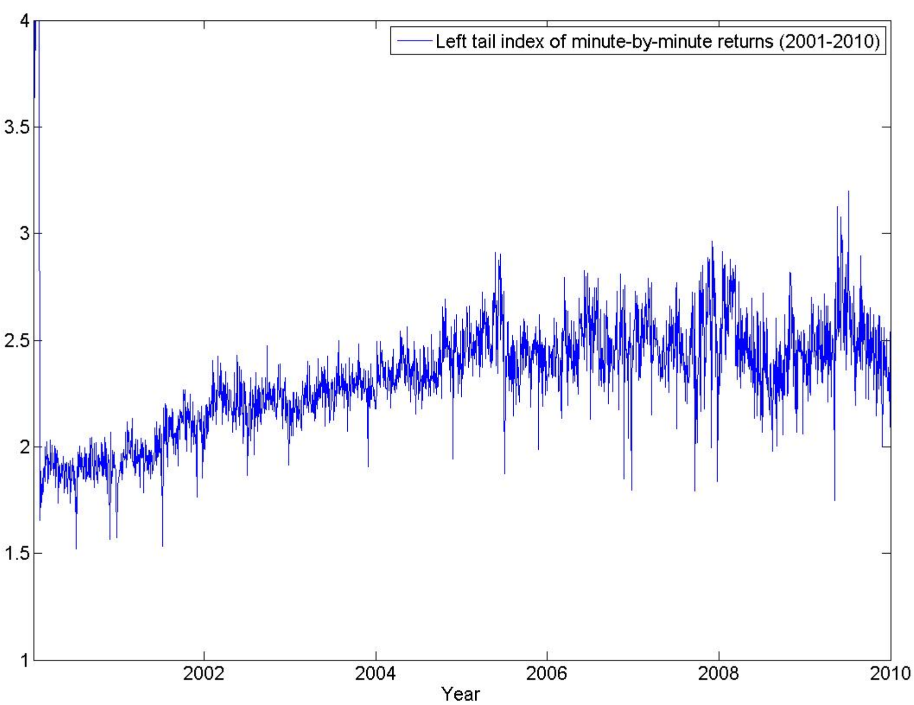

Figure 12: High Frequency-based Tail Index in Liquidity: 2001-2010

The figure displays liquidity tail indices computed from intraday data. The intraday-based liquidity tail index is computed in two steps. First we compute the tail index for each stock from minute-by-minute liquidity (effective spread, absolute spread, and relative spread), using NYSE-TAQ stocks. Second, we average the tail index across all stocks to obtain a market liquidity tail index, reported in the figure. The data comprise NYSE, AMEX and NASDAQ stocks with prices between $5 and $1000. The sample period is 2001 to 2010.

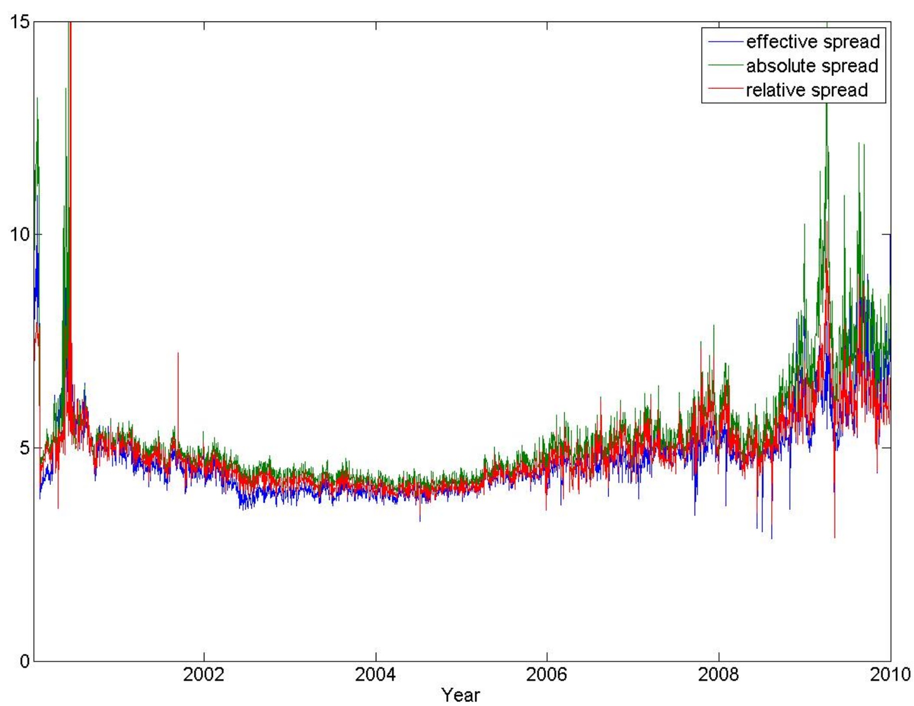

Figure 13: Liquidity Tail Index from Daily and Intraday Data

The figure shows liquidity tail indices computed from both daily and intraday data. Both measures are estimated using the method of Hill (1975), from equation (2). The daily-based tail index is computed using the cross section of Amihud (2002) liquidity each day. The intraday-based liquidity tail index is computed in two steps. First we compute the tail index for each stock from minute-by-minute liquidity (net order flow), using NYSE-TAQ stocks. Second, we average the tail index across all stocks to obtain a market liquidity tail index, reported in the figure. The data comprise NYSE, AMEX and NASDAQ stocks with prices between $\$5$ and $\$1000$. The sample period is 1993 to 2010.

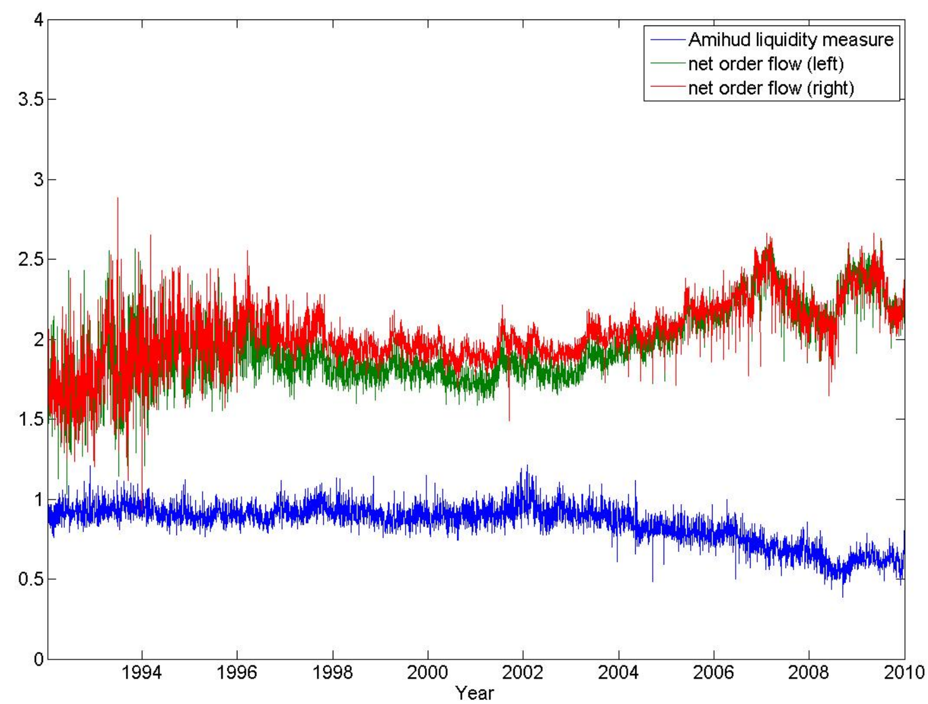

Table 19: Properties of Intraday Tail Index for Returns and Liquidity

The table presents correlations of the tail index, computed for both intraday returns and intraday liquidity. Liquidity is measured in 4 different ways: net order flow, effective spread, absolute spread, and relative spread. The tail index is estimated in all cases using the method of Hill (1975), as in equation (2). All tail indices refer to the *left tail*, unless otherwise specified. The intraday-based return (liquidity) tail index is computed in two steps. First we compute the tail index for each stock from minute-by-minute returns (liquidity measures), using NYSE-TAQ stocks. Second, we average the tail index across all stocks to obtain a market return (liquidity) tail index, reported in the figure. Data comprise TAQ firms with prices between \$5 and \$1000, and include firms listed on NYSE, AMEX and NASDAQ during the sample period, January 1993 through December 2010. P-values are presented in parentheses.

Panel A: Pearson Correlation Coefficients

	Net Order Flow (Left Tail)	Net Order Flow	Effective Spread	Absolute Spread	Relative Spread	Return (Left Tail)
Net Order Flow (Left Tail)	1	0.7626	-0.3045	-0.2925	0.0080	-0.3326
		($<$ .0001)	($<$ .0001)	($<$ .0001)	(0.5938)	($<$ .0001)
Net Order Flow		1	-0.4139	-0.3733	0.0255	-0.4243
			($<$ .0001)	($<$ .0001)	(0.0876)	($<$ .0001)
Effective Spread			1	0.6478	0.0177	0.7896
				($<$ .0001)	(0.2366)	($<$ .0001)
Absolute Spread				1	-0.0041	0.7562
					(0.7816)	($<$ .0001)
Relative Spread					1	0.0553
						(0.0002)

Panel B: Spearman Correlation Coefficients

	Net Order Flow (Left Tail)	Net Order Flow	Effective Spread	Absolute Spread	Relative Spread	Return (Left Tail)
Net Order Flow (Left Tail)	1	0.7528	-0.2451	-0.2227	-0.2507	-0.1477
		($<$ .0001)	($<$ .0001)	($<$ .0001)	($<$ .0001)	($<$ .0001)
Net Order Flow		1	-0.2706	-0.2476	-0.2738	-0.1823
			($<$ .0001)	($<$ .0001)	($<$ .0001)	($<$ .0001)
Effective Spread			1	0.9703	0.9778	0.8308
				($<$ .0001)	($<$ .0001)	($<$ .0001)
Absolute Spread				1	0.9798	0.8270
					($<$ .0001)	($<$ .0001)
Relative Spread					1	0.8277
						($<$ .0001)